

Crystals in High Energy Physics

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- Discovery of the channeling phenomenon
- Geometrical optics method in the quantum theory of High energy charged particles scattering in external field
- Quantum and classical effects in scattering
- Beam deflection by bent crystals
-

Discovery of the channeling phenomenon

X-Rays discovery (1895)

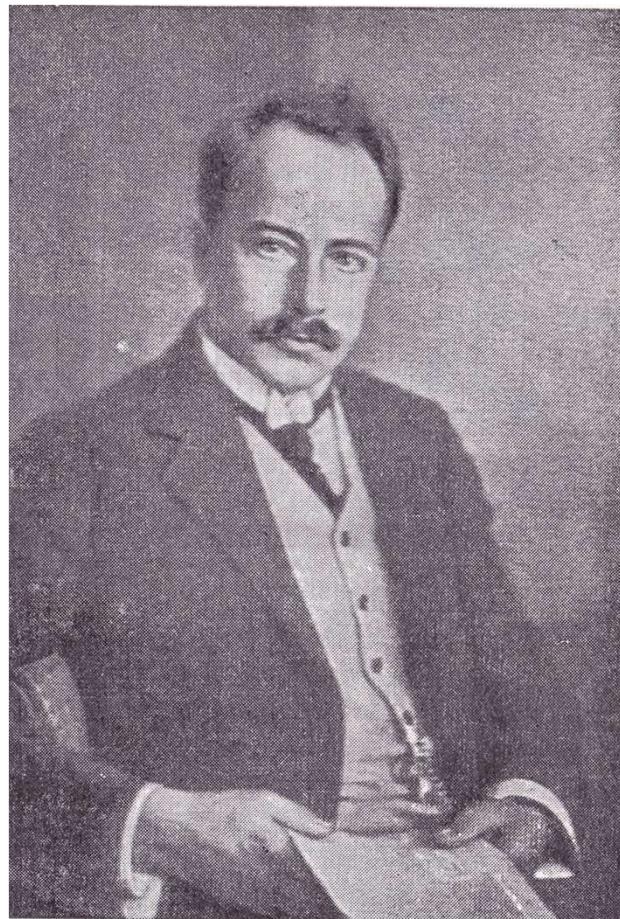
W.K. Röntgen



1901 Nobel Prize

X-Ray Diffraction Analysis

M. von Laue



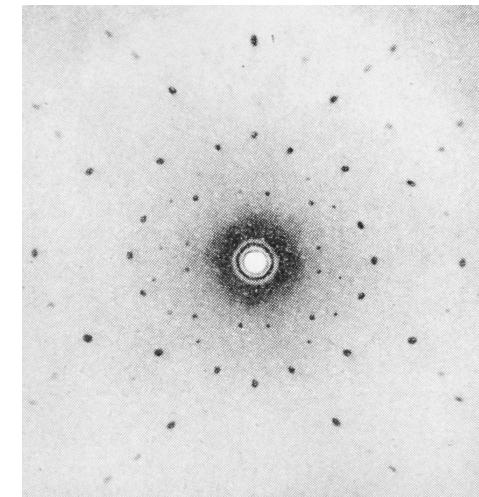
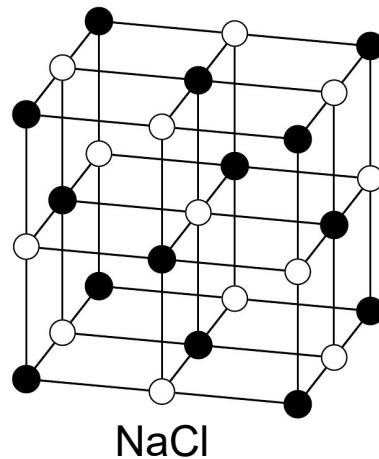
1914 Nobel Prize

Interaction of particles and waves with crystals

Problem: *the nature of the X-rays – particles or waves?*



M. von Laue

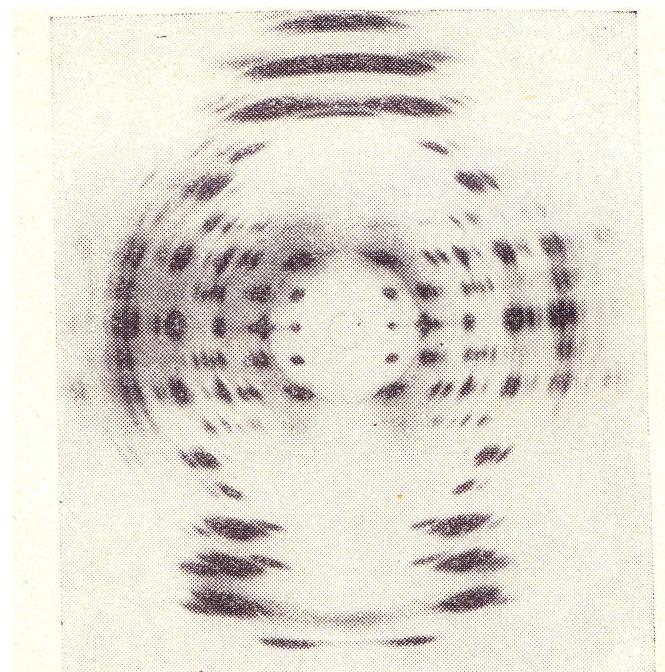
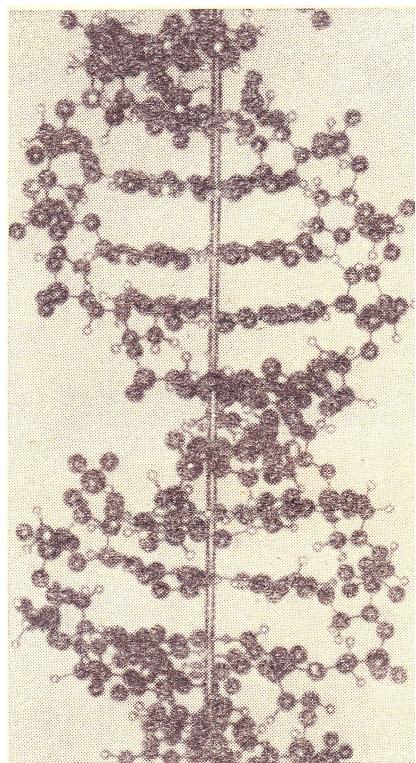


From: John C. Kendrew. The Thread of Life. London, G. Bell&Sons LTD, 1964.

$$\lambda = \frac{\hbar}{p} \geq a$$

H. Bethe, Ann. Phys. 4 (1928) 55,87.
F. Bloch, Z. Phys Bd 81 (1933) 363.

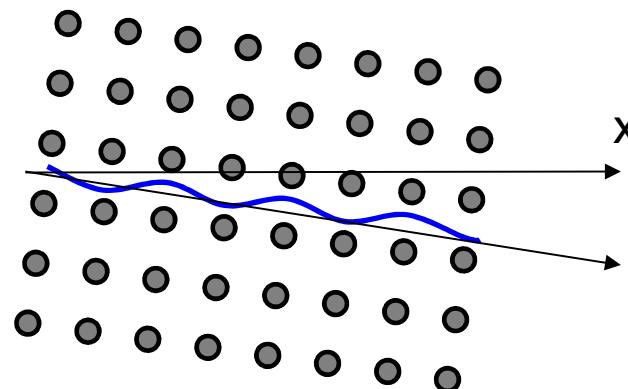
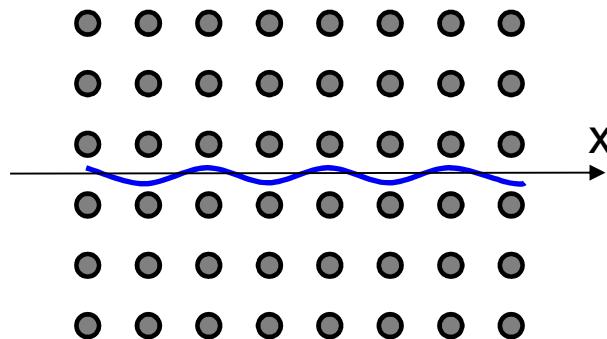
X-Ray Diffraction Analysis of DNA



Nature of X-Rays

Open channels for particles

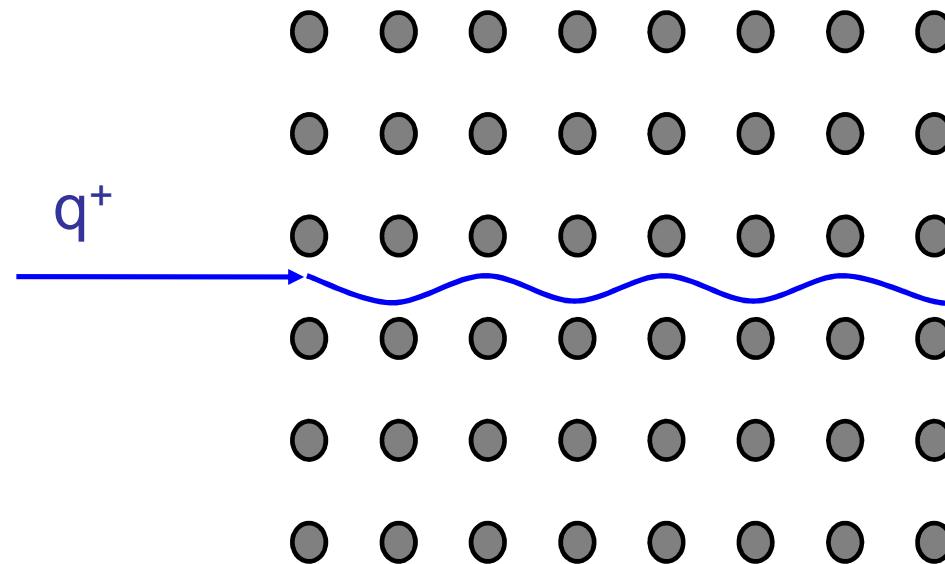
G. Bragg, L. Bragg (1912)



Bragg reflection: 1915 Nobel Prize

Open Channels for Charged Particles

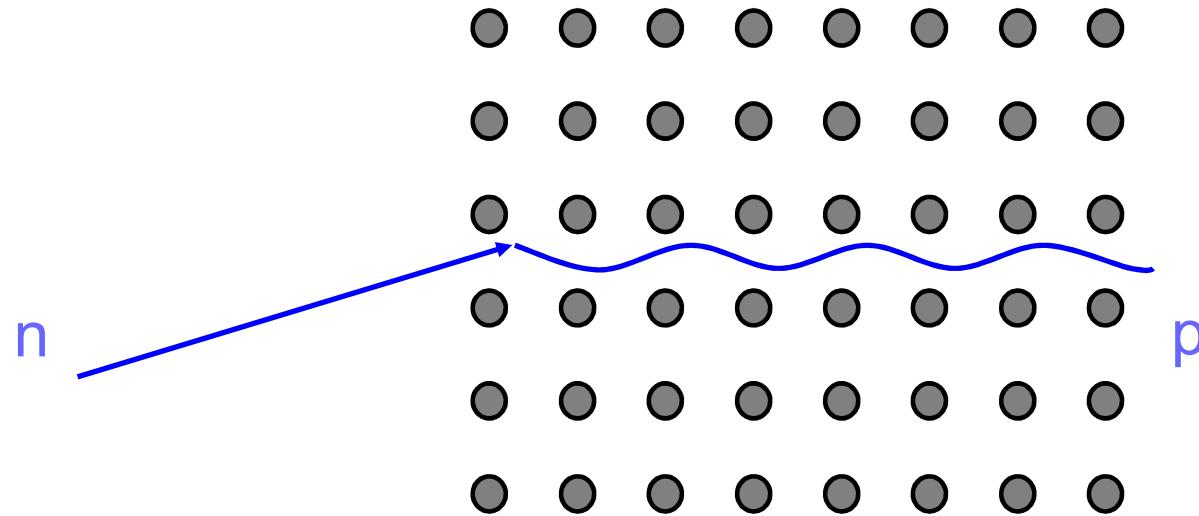
J. Stark (1912)



1919 Nobel Prize

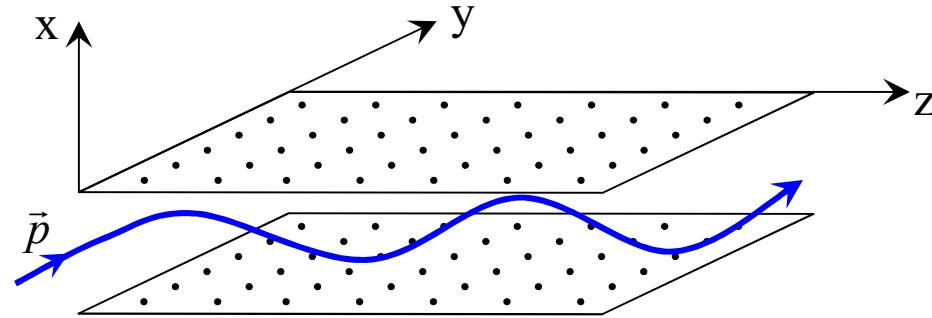
Protons Channeling

M. Robinson, O. Oen (1961)



Phenomenon of Planar Channeling

J.Lindhard (1965)

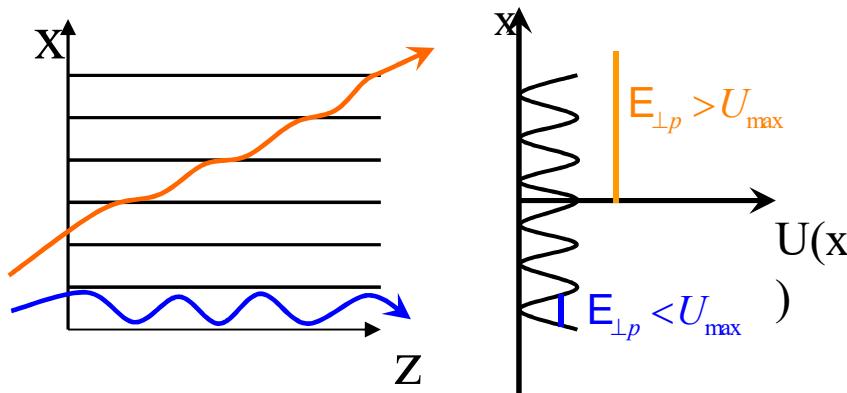


$$E_{\perp} = \frac{E\psi_c^2}{2} = U_{\max} \quad \Leftrightarrow \quad \boxed{\psi_c \sim \sqrt{2U_{\max}/E}}$$

$$p_z = \text{const} \approx p$$

$$p_y = \text{const} \approx 0$$

$$\begin{aligned} \ddot{x} &= -\frac{1}{E} \frac{\partial}{\partial x} U(x) \\ E_{\perp} &= \frac{E \dot{x}^2}{2} + U(x) \end{aligned}$$



Quantum consideration

$$\begin{aligned} \psi &= e^{i(pz - \varepsilon t)} \varphi(x, t) \\ i\hbar \partial_t \varphi &= \left(-\frac{\hbar^2}{2\varepsilon} \frac{\partial^2}{\partial x^2} + U(x) \right) \varphi(x, t) \end{aligned}$$

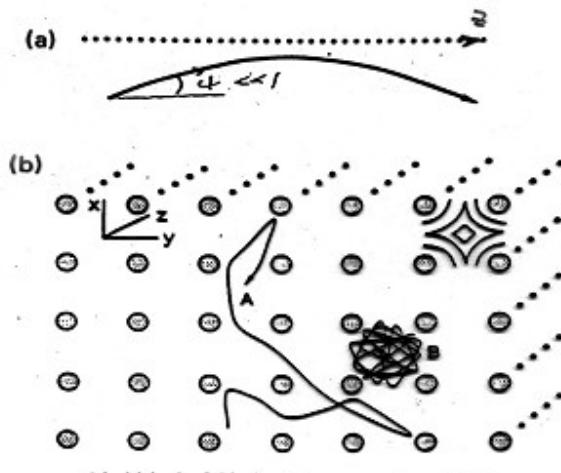
$$\boxed{n_{\text{levels}} \sim \sqrt{E_{\text{MeV}}}}$$

Phenomenon of Above Barrier Motion: A. Akhiezer, N. Shul'ga (1978)

Axial Channeling and Above-Barrier Motion (continuous string potential)

$$\frac{d\mathbf{p}}{dt} = -\nabla U(\mathbf{r})$$

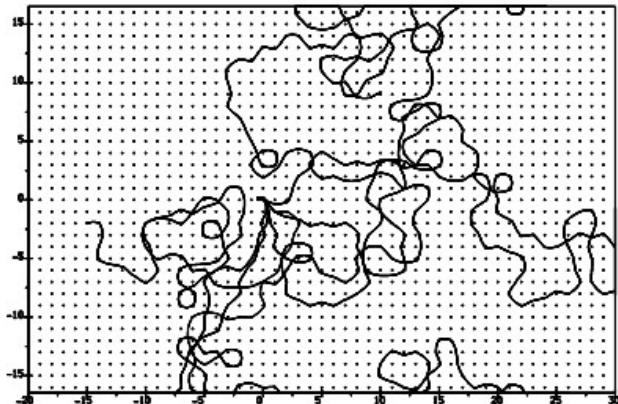
$$U(\mathbf{r}) \rightarrow U(x, y) = \frac{1}{L} \int_0^L dz \sum_n u(\mathbf{r} - \mathbf{r}_n)$$



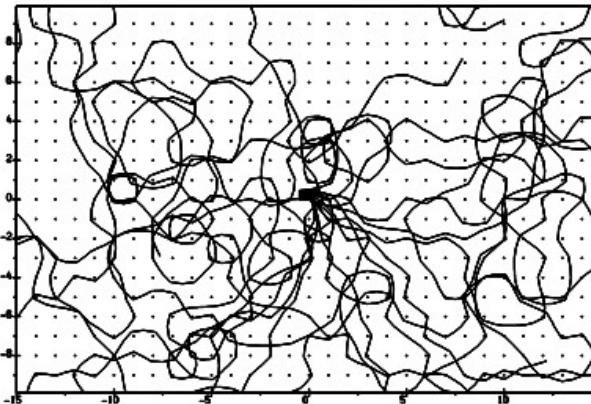
$$\frac{d\mathbf{p}}{dt} = -\nabla U(x, y) \quad \rightarrow \quad \begin{cases} p_z = \text{const} \gg p_\perp \\ \ddot{\vec{\rho}} = -\frac{1}{\epsilon} \nabla U_\perp(x, y) \end{cases}$$

Dynamical Chaos at Multiple Scattering for e^-

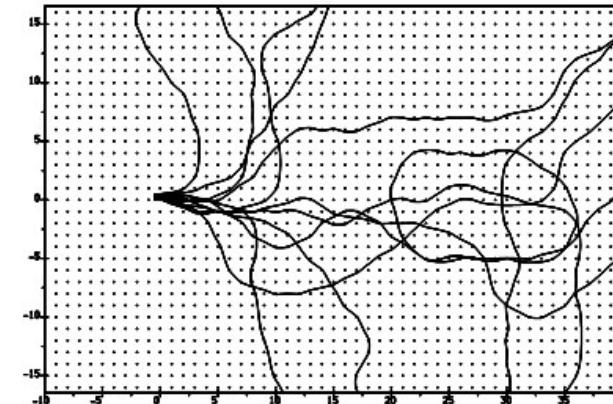
$$z = \psi/\psi_c$$



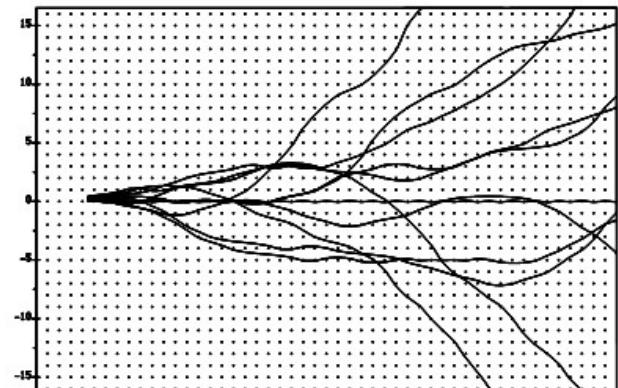
$Z=0.5$



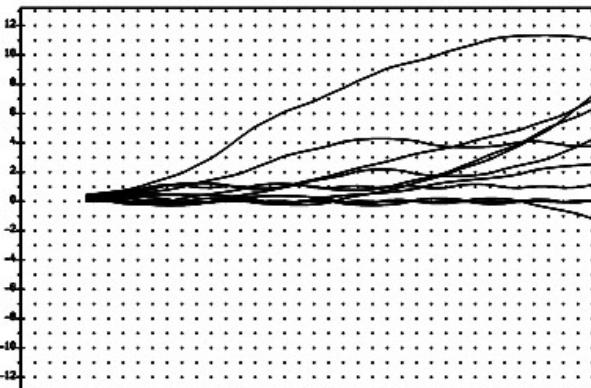
$Z=0.7$



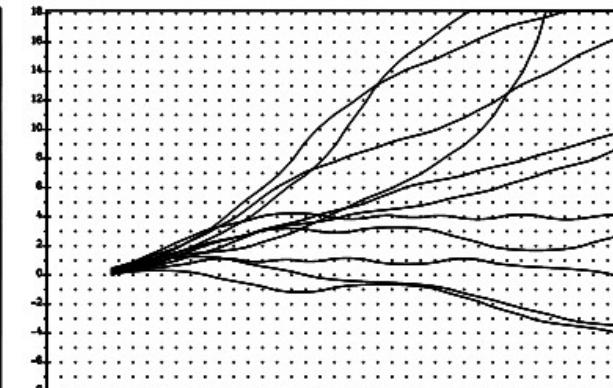
$Z=1.5$



$Z=1$



$Z=2, \alpha = 6^0$



$Z=2, \alpha = 15^0$

Geometrical Optics Method in the Quantum Theory of High Energy Charged Particles Scattering in External Field

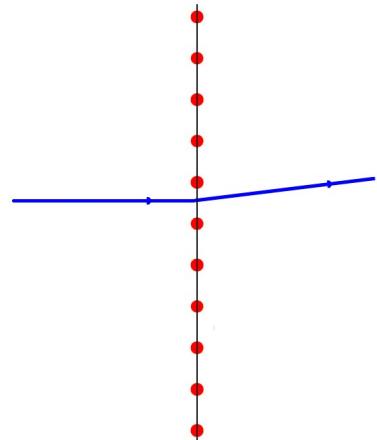
N.F. Shul'ga, S.N. Shulga

N.F. Shul'ga, S.N. Shulga. Phys. Lett. B 769 (2017) 141,

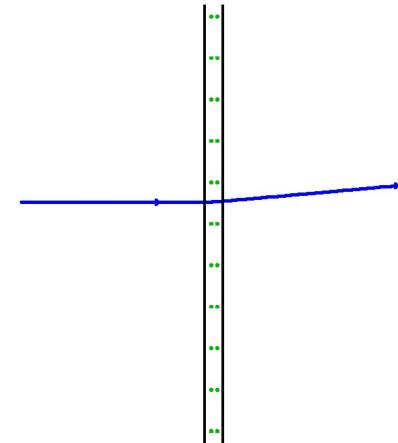
S.N. Shulga, N.F. Shul'ga, S. Barsuk, I. Chaikovska, R. Chehab. NIM B 402 (2017) 16.

N.F. Shul'ga, S.N. Shulga. Phys. Lett. B 791 (2019) 225

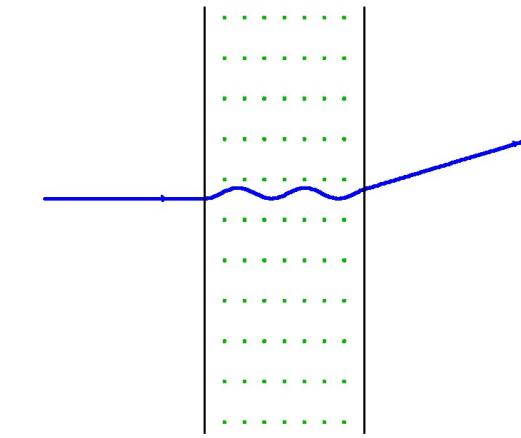
Scattering in ultrathin crystals



Graphene



Ultrathin crystal
coherent effects



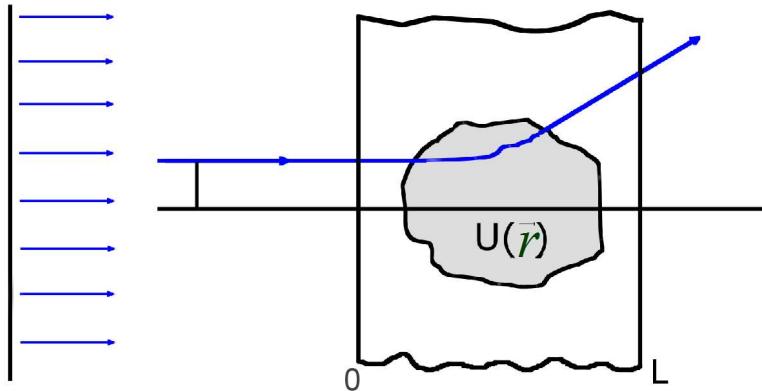
Channeling

Classical Mechanics

$$\ddot{\mathbf{p}} = -\frac{1}{\varepsilon} \nabla U(x, y) \quad \backslash \quad d\sigma_{cl}(\vartheta) = \sum_n d^2 b_n(\vartheta) = \sum_n \left. \frac{1}{|\partial \vartheta / \partial \mathbf{b}|_n} \right|_{\mathbf{b}=\mathbf{b}_n(\vartheta)} d^2 \vartheta$$

Gauss Theorem in Quantum Scattering Theory

N. Bondarenko, N. Shul'ga Phys. Lett. B 427 (1998) 114



$$\psi = \varphi(\mathbf{r}) e^{i\mathbf{p}\cdot\mathbf{r}} u_p$$

$$\mathbf{q} = \mathbf{p} - \mathbf{p}'$$

$$a(\vartheta) = -\frac{1}{4\pi} \int_V d^3 r e^{-i\mathbf{p}'\cdot\mathbf{r}} \bar{u}' \gamma_0 U(\mathbf{r}) \psi(\mathbf{r}) = -\frac{1}{4\pi} \int_V d^3 r \operatorname{div} [\bar{u}' \gamma \psi(\mathbf{r}) e^{-i\mathbf{p}'\cdot\mathbf{r}}] =$$

$$= -\frac{i}{4\pi} \oint d\mathbf{S} \bar{u}' \gamma \psi(\mathbf{r}) e^{-i\mathbf{p}'\cdot\mathbf{r}} = -\frac{i}{4\pi\hbar} \oint d^2 \rho e^{i\mathbf{q}\cdot\mathbf{r}} \bar{u}' \gamma_z \psi(\mathbf{p}, z) e^{-i\mathbf{p}'\cdot\mathbf{r}} \Big|_{z=0}^{z=L}$$

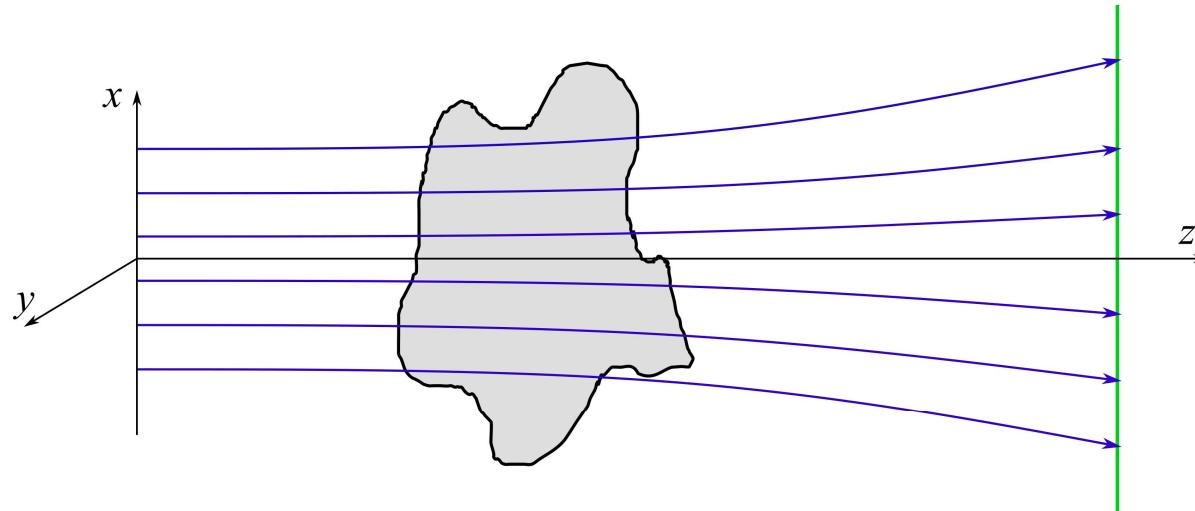
$$\frac{d\sigma_q}{do} = |a(\vartheta)|^2$$

$$\sigma_{tot} = \frac{4\pi\hbar}{p} \operatorname{Im} a(\vartheta) \Big|_{\vartheta=0}$$

Geometrical Optics Method (ray optics)

$$\Delta u + k_0^2 n^2(\mathbf{r}) u = 0$$

$$k_0^2 n^2(\mathbf{r}) = 2m(\varepsilon - U(\mathbf{r}))/\hbar^2$$



$$u(\mathbf{r}) = \sqrt{A_0} e^{ik_0 \psi(\mathbf{r})}$$

$$\begin{cases} (\nabla \psi)^2 = n^2 \\ \nabla(A_0 \nabla \psi) = 0 \end{cases}$$

$$\frac{d^2 \mathbf{r}}{d\tau^2} = \frac{1}{2} \nabla n^2(\mathbf{r})$$

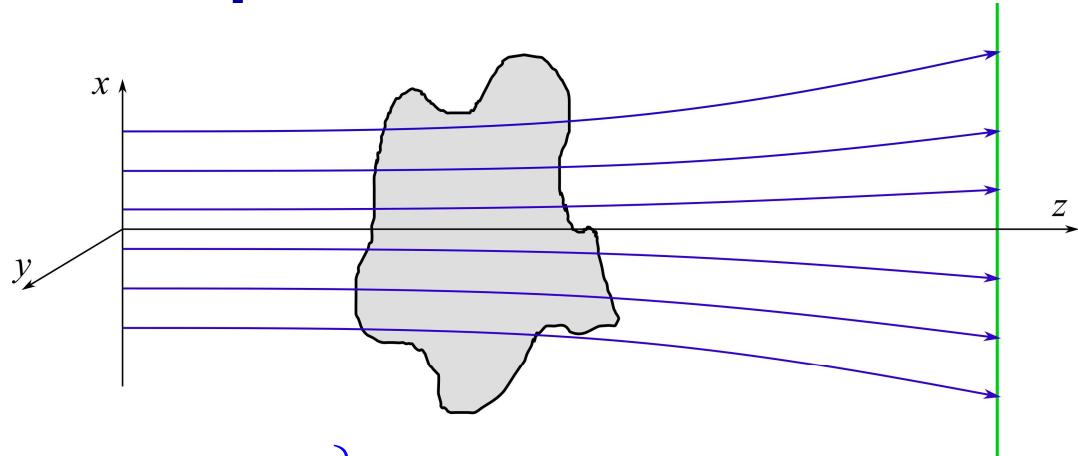
V. Maslov, M. Fedoriuk *Semi-classical approximation in quantum mechanics*, D.R., Holland, 1981

V. Arnold, *Mathematical Methods in Classical Mechanics*, Springer, NY, 1989

16

Y. Kravtsov, Y. Orlov, *Geometrical Optics of Inhomogeneous Media*, Springer, Berlin, 2011

Geometrical Optics Method



$$u(\mathbf{r}) = \frac{1}{\sqrt{|D|}} \exp \left\{ ik_0 \int_0^\tau d\tau' n^2(\mathbf{r}(\tau')) - i \frac{\pi}{2} \mu \right\}$$

(μ – Maslov-Morse index)

$$\frac{d^2 \mathbf{r}}{d\tau^2} = \frac{1}{2} \nabla n^2(\mathbf{r})$$

$$D = \frac{\partial(x, y, z)}{\partial(x_0, y_0, z_0)} = \begin{vmatrix} \frac{\partial x}{\partial x_0} & \frac{\partial x}{\partial y_0} & \frac{\partial x}{\partial z_0} \\ \frac{\partial y}{\partial x_0} & \frac{\partial y}{\partial y_0} & \frac{\partial y}{\partial z_0} \\ \frac{\partial z}{\partial x_0} & \frac{\partial z}{\partial y_0} & \frac{\partial z}{\partial z_0} \end{vmatrix}$$

Semiclassical approximation for the wave function

$$\left[(\varepsilon - U)^2 - (i\hbar \nabla)^2 - m^2 + i\hbar \gamma_0 \gamma \nabla U \right] \psi = 0$$

$$\psi^{WKB}(\mathbf{p}, z) = \sqrt{f(\mathbf{p}, z)} \exp\left(\frac{i}{\hbar} S(\mathbf{p}, z)\right), \quad S = \mathbf{p}\mathbf{r} + \chi(\mathbf{r})$$

$$\begin{cases} (\varepsilon - U(\mathbf{r}))^2 = (\nabla S)^2 + m^2 \\ \nabla S \cdot \nabla f + (\nabla^2 S) f = 0 \end{cases}$$

$$-v \partial_z \chi = U_c(\mathbf{p}) + \frac{1}{2\varepsilon} (\nabla_{\perp} \chi(\mathbf{p}, z))^2$$

Geometrical optics (ray optics)

N.F. Shul'ga, S.N. Shulga. Phys. Lett. B 791 (2019) 225

$$\psi(\rho(\mathbf{b}, z), z) = \frac{1}{\sqrt{|D|}} \exp \left\{ ipz + i\chi(\rho(\mathbf{b}, z), z) - i\frac{\pi}{2}\mu \right\}$$

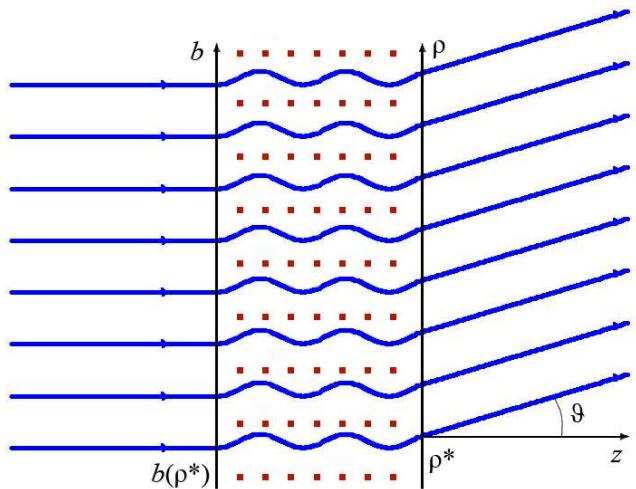
(μ – Maslov-Morse index)

$$\frac{d^2\rho}{dz^2} = -\frac{c^2}{\varepsilon} \frac{\partial}{\partial \rho} U(\rho, z) \quad \rightarrow \quad \rho = \rho(\mathbf{b}, z) \quad \mathbf{b} = \rho \Big|_{z=0}$$

$$\chi(\rho(\mathbf{b}, z), z) = -\frac{1}{\nu} \int_0^z dz' \left[2U_c(\rho(\mathbf{b}, z'), z') - \varepsilon_\perp \right]$$

$$D = \frac{\partial(x, y, z)}{\partial(b_x, b_y, \tau)} = \det \begin{pmatrix} \partial_{b_x} x & \partial_{b_y} x & \partial_\tau x \\ \partial_{b_x} y & \partial_{b_y} y & \partial_\tau y \\ \partial_{b_x} z & \partial_{b_y} z & \partial_\tau z \end{pmatrix}$$

Wave function in geometrical optics approximation for plane channeled positrons



$$\ddot{x} = -\frac{1}{\varepsilon} \frac{\partial}{\partial x} U_c(x)$$

$$U_c(x) = U_0 \frac{x^2}{(a/2)^2}, \quad |x| \leq a/2$$

$$\ddot{x} + \Omega^2 x = 0 \quad \Omega = \sqrt{\frac{8U_0}{\varepsilon a^2}} = \frac{2\theta_p}{a}$$

$$x = b \cos \Omega z$$

$$\chi(x(b,z), z) = -\frac{1}{\nu} \frac{2b^2}{a^2} \frac{U_0}{\Omega} \sin 2\Omega z$$

$$D = \cos \Omega z$$

$$\boxed{\psi(x, z) = \frac{1}{\sqrt{\cos \Omega z}} \exp \left\{ i p z - i \frac{2U_0}{\Omega} \frac{b^2}{a^2} \sin 2\Omega z \right\}}$$

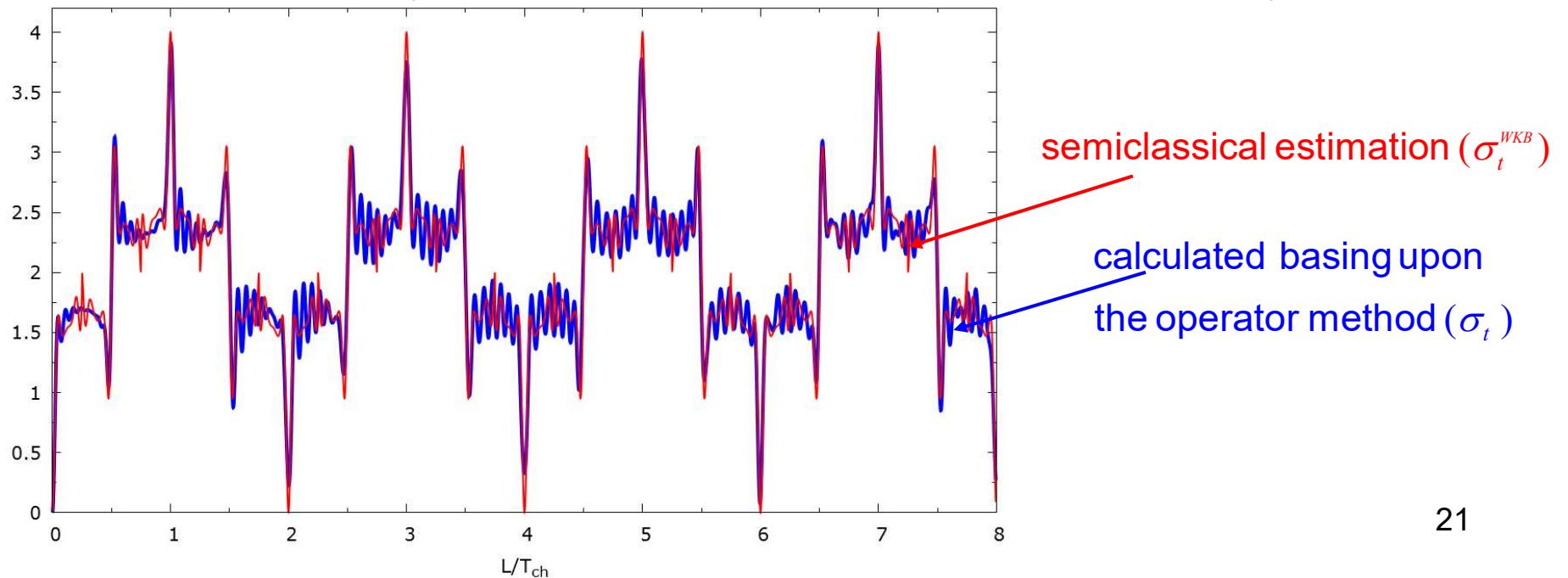
Total scattering cross-section

(S. Shulga, N. Shul'ga Phys. Lett. B 791 (2019) 225)

$$\sigma_t = \frac{4\pi\hbar}{p} \text{Im } a \Big|_{\theta=0} = 2L_y L_x \left(1 - \frac{1}{a} \text{Re} \int_{x_{\min}}^{x_{\max}} dx(b, L) \varphi(x(b, L), L) \right)$$

$$dx = \left| \frac{dx}{db} \right| db = |\cos \Omega L| db$$

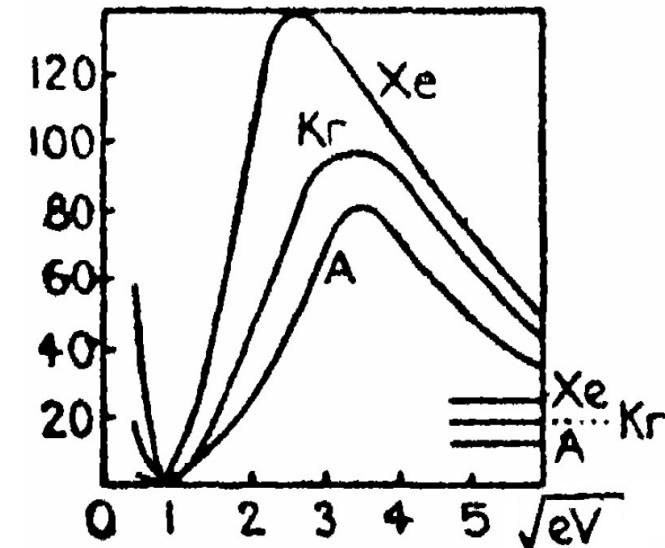
$$\sigma_t^{WKB} = 2L_y L_x \left\{ 1 - \frac{1}{a} \text{Re} \int_{-a/2}^{a/2} db |\cos \Omega L|^{1/2} e^{-\frac{i}{\hbar v} \frac{2U_0}{\Omega} \frac{b^2}{a^2} \sin 2\Omega L - i \frac{\pi}{2} \mu} \right\}$$



Ramsauer-Townsend effect (scattering of electrons on Xe, Kr, Ar atoms at $\epsilon \sim 0.7$ eV)

P. Ramsauer, Ann. d. Phys. 64 (1921) 513,
J. Townsend, V. Bailey. Phil. Mag. (1922) 593.

atomic field	—	present
force	—	present
scattering	—	absent

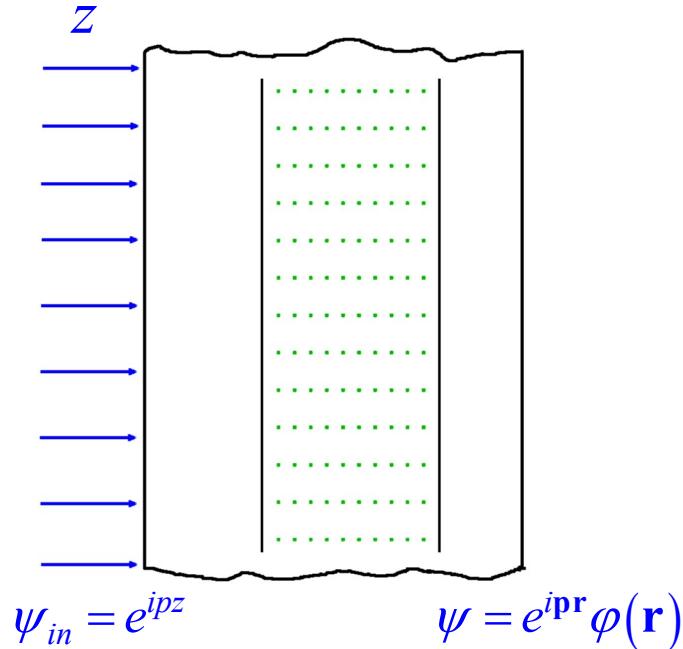


From: N.Mott, H.Massey. The theory of atomic collisions (fig.89), Oxford, CP (1965)



N. Bohr was delighted with this effect, because it is a quantum (interference) phenomenon

Operator method



wave function

$$\psi = e^{i(pz - \varepsilon t)} \varphi(\mathbf{p}, z)$$

$$i\hbar v \partial_z \varphi(\mathbf{p}, z) = \left(\frac{\hat{\mathbf{p}}_\perp^2}{2\varepsilon} + U(\mathbf{p}) \right) \varphi(\mathbf{p}, z) = \\ = (\hat{H}_0 + U(\mathbf{p})) \varphi$$

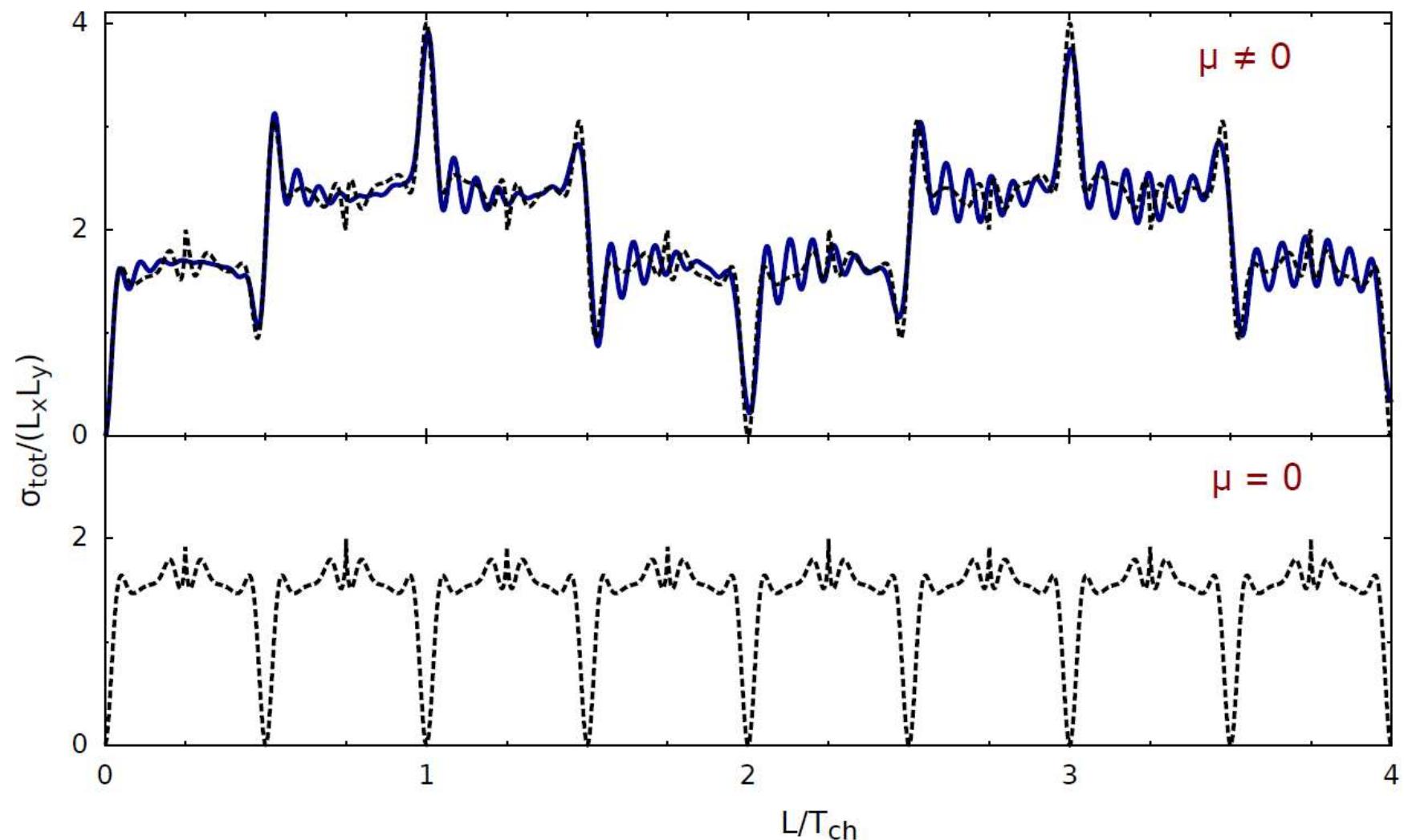
$$\varphi(\mathbf{p}, z + \Delta z) = e^{-\frac{i}{\hbar}(\hat{H}_0 + U(\mathbf{p}))\Delta z} \varphi(\mathbf{p}, z)$$

+ iteration procedure

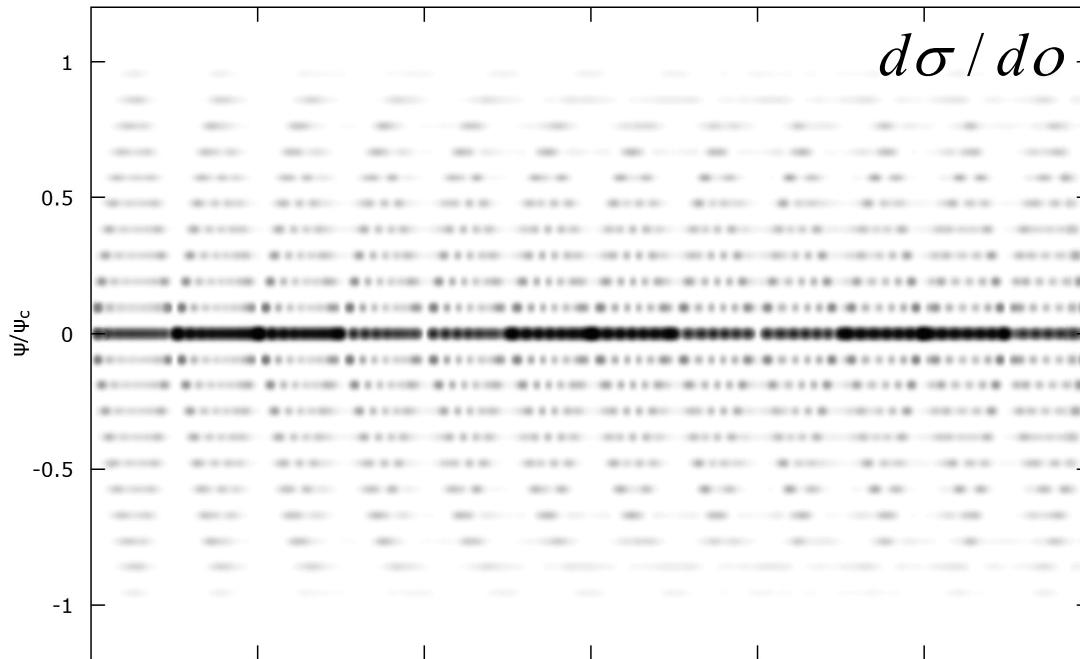
M. Feit, J. Fleck et al., J. Comput. Phys. 47 (1982) 412,
 S. Dabagov, L. Ognev, NIM B 30 (1988) 185,
 N. Shul'ga, S. Shul'ga Phys. Lett. B 769 (2017) 141.

Total scattering cross-section and Maslov-Morse index μ

$e^+, \varepsilon = 100 \text{ MeV}$

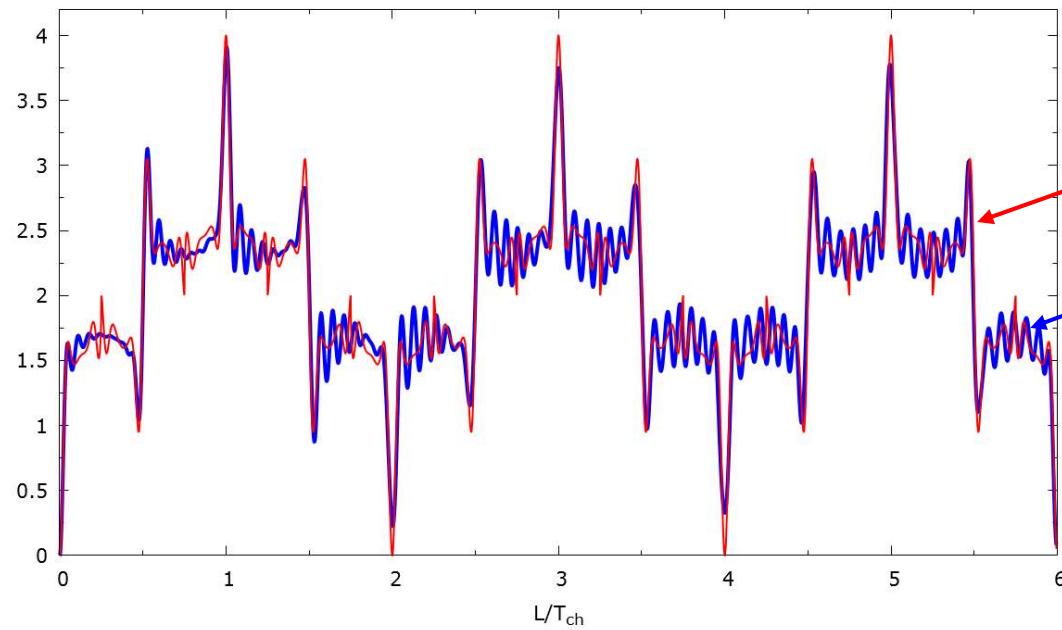


Differential and total scattering cross-sections

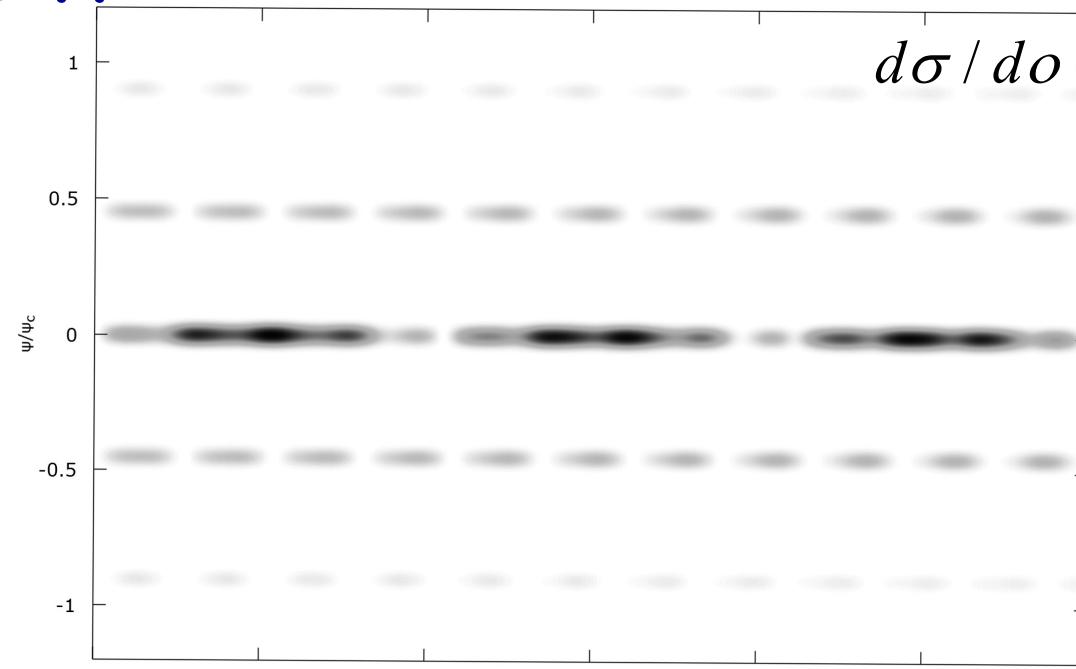


positrons
 $\varepsilon = 100 \text{ MeV}$

$$U_c = U_0 \left(\frac{x}{d/2} \right)^2$$

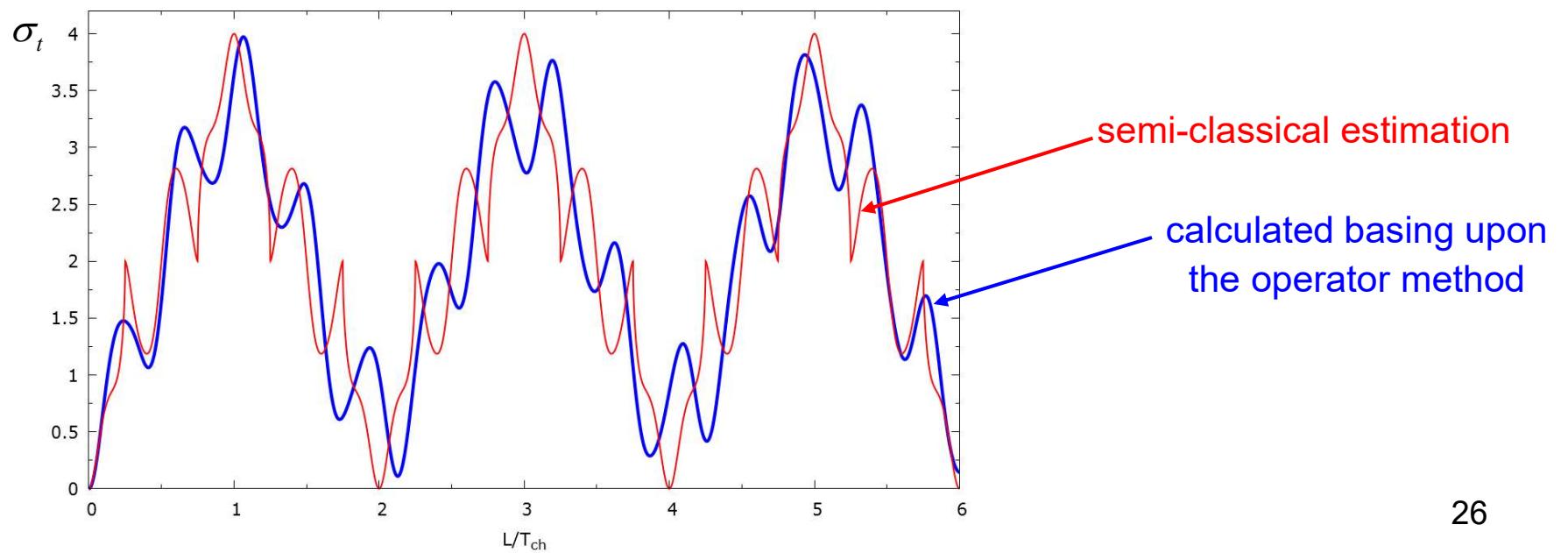


Differential and total scattering cross-sections

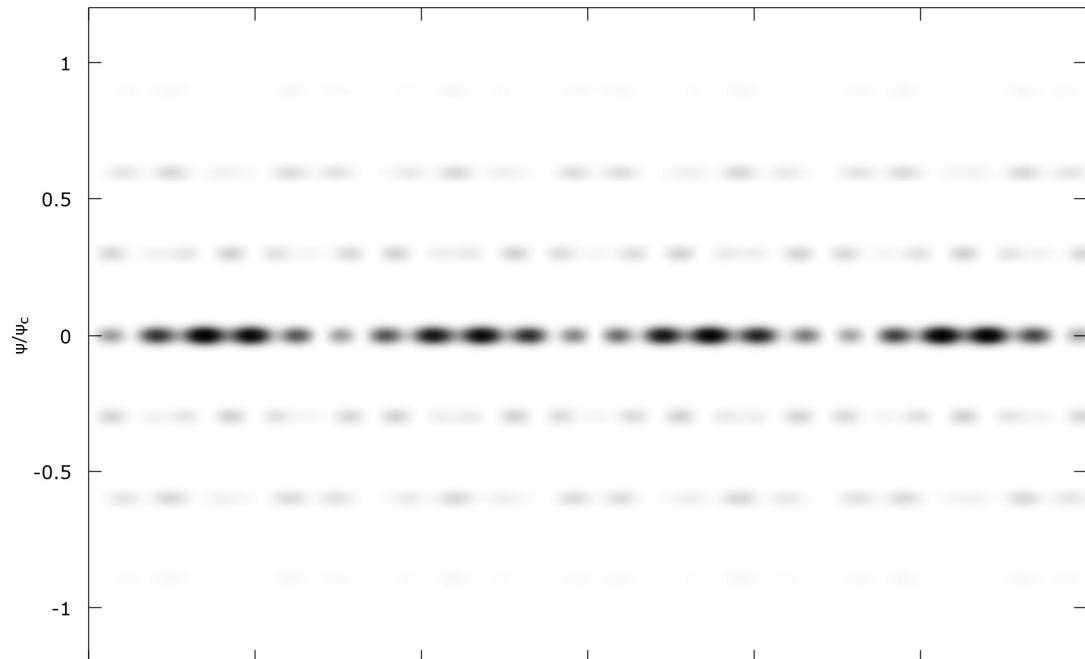


e^+
 $\varepsilon = 4 \text{ MeV}$

$$U_c = U_0 \left(\frac{x}{d/2} \right)^2$$

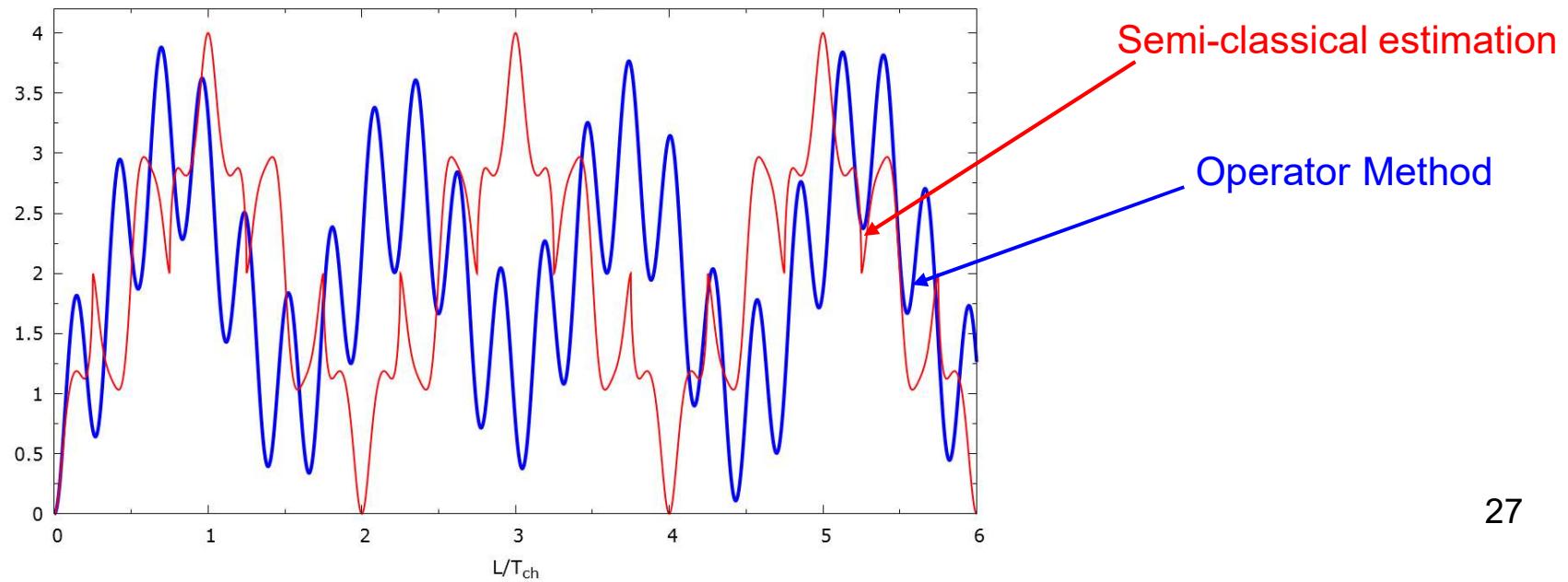


Differential and total scattering cross-sections

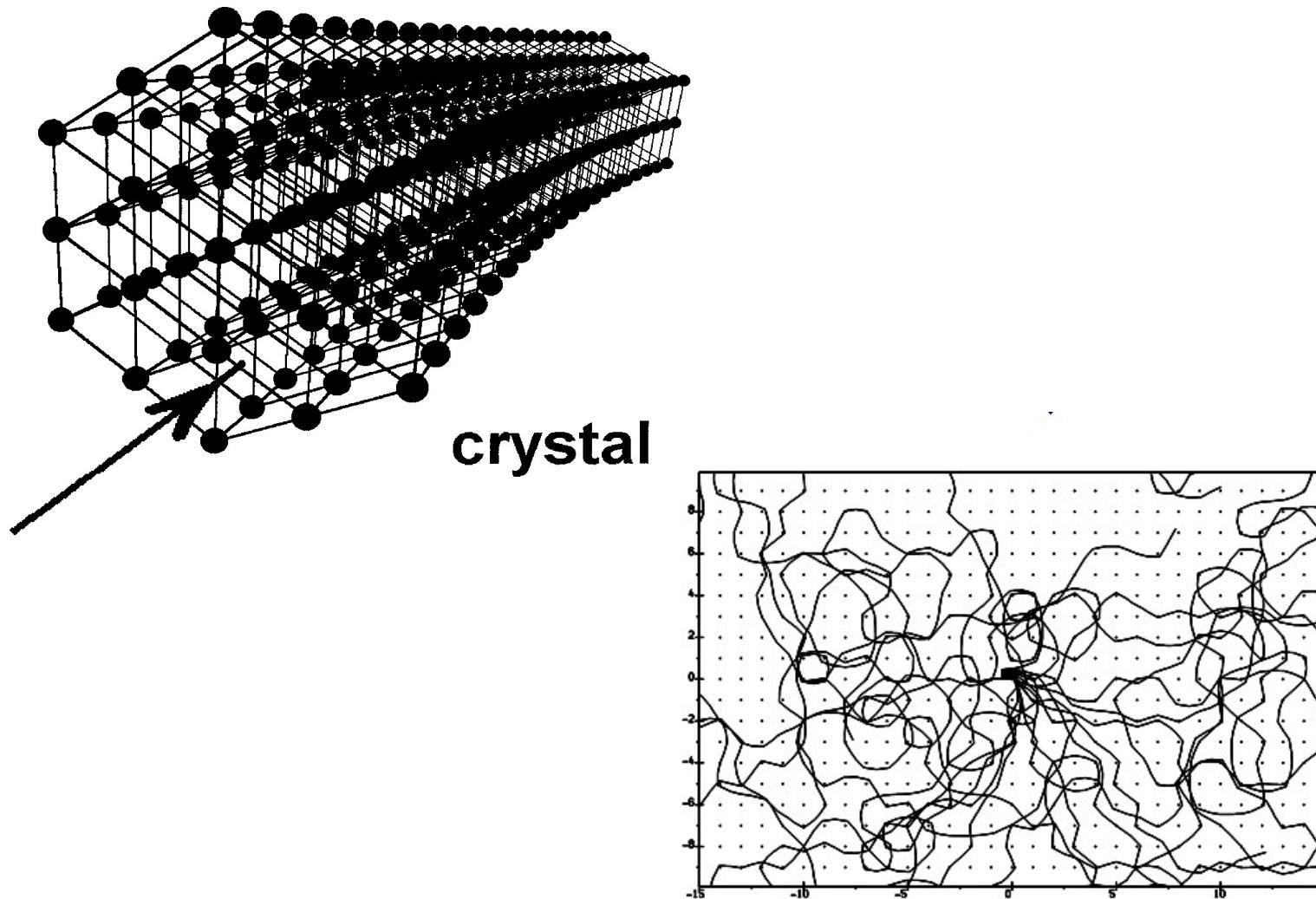


e^-
 $\varepsilon = 9.8 \text{ MeV}$

$$U_c = -U_0 \left(\frac{x}{d/2} \right)^2$$

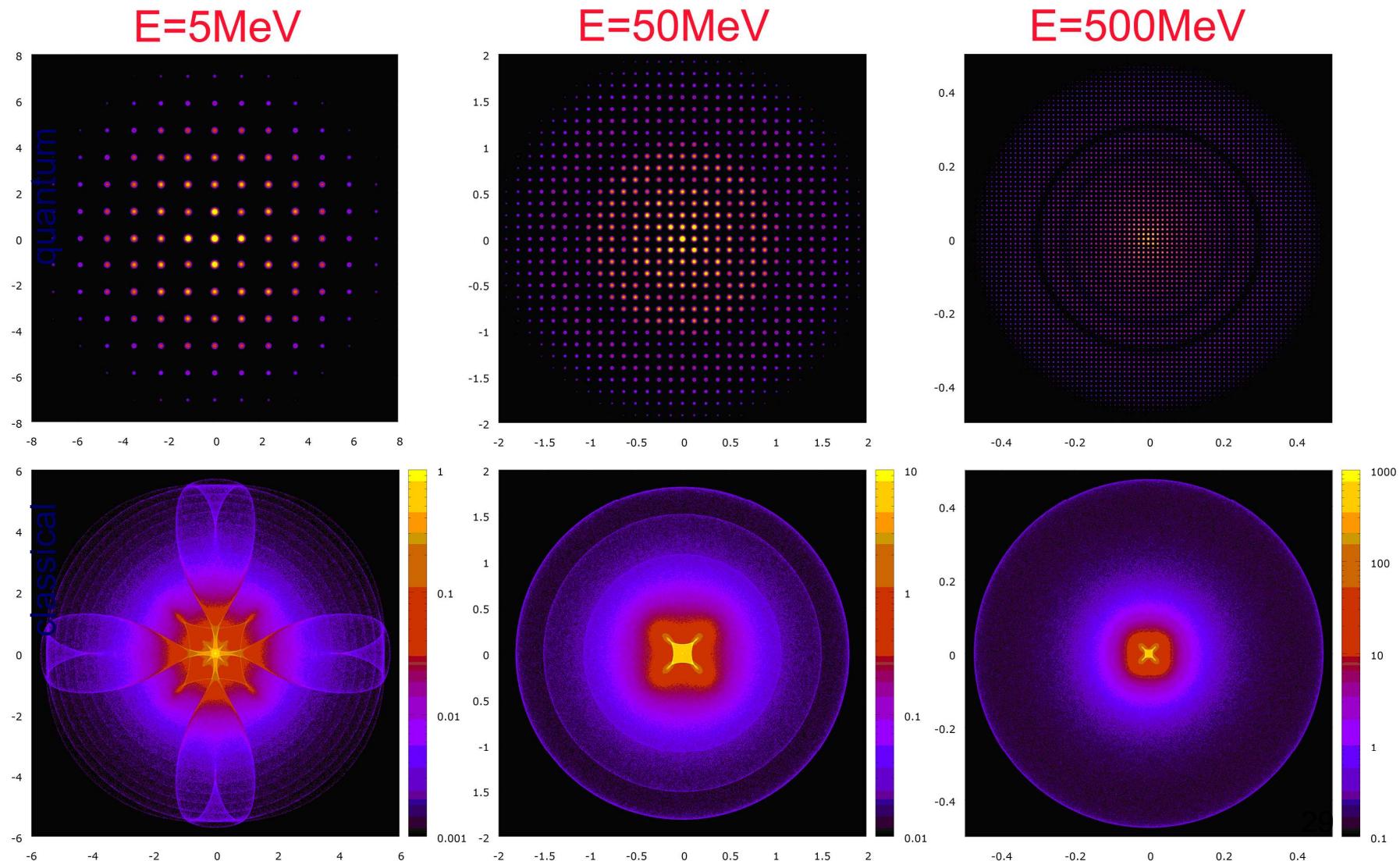


Dynamical Chaos at Multiple Scattering on atomic strings

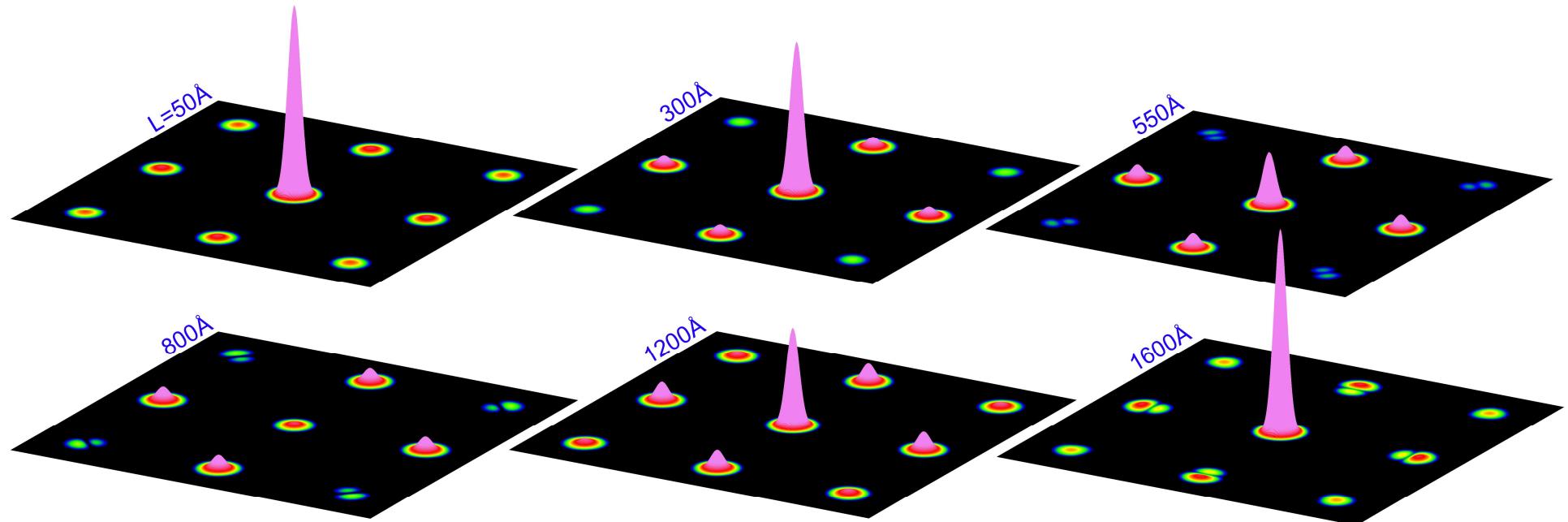


Quantum and classical angular distributions of electrons, 1000Å Si <100>

N. Shul'ga, S. Shul'ga Phys. Lett. B 769 (2017) 141

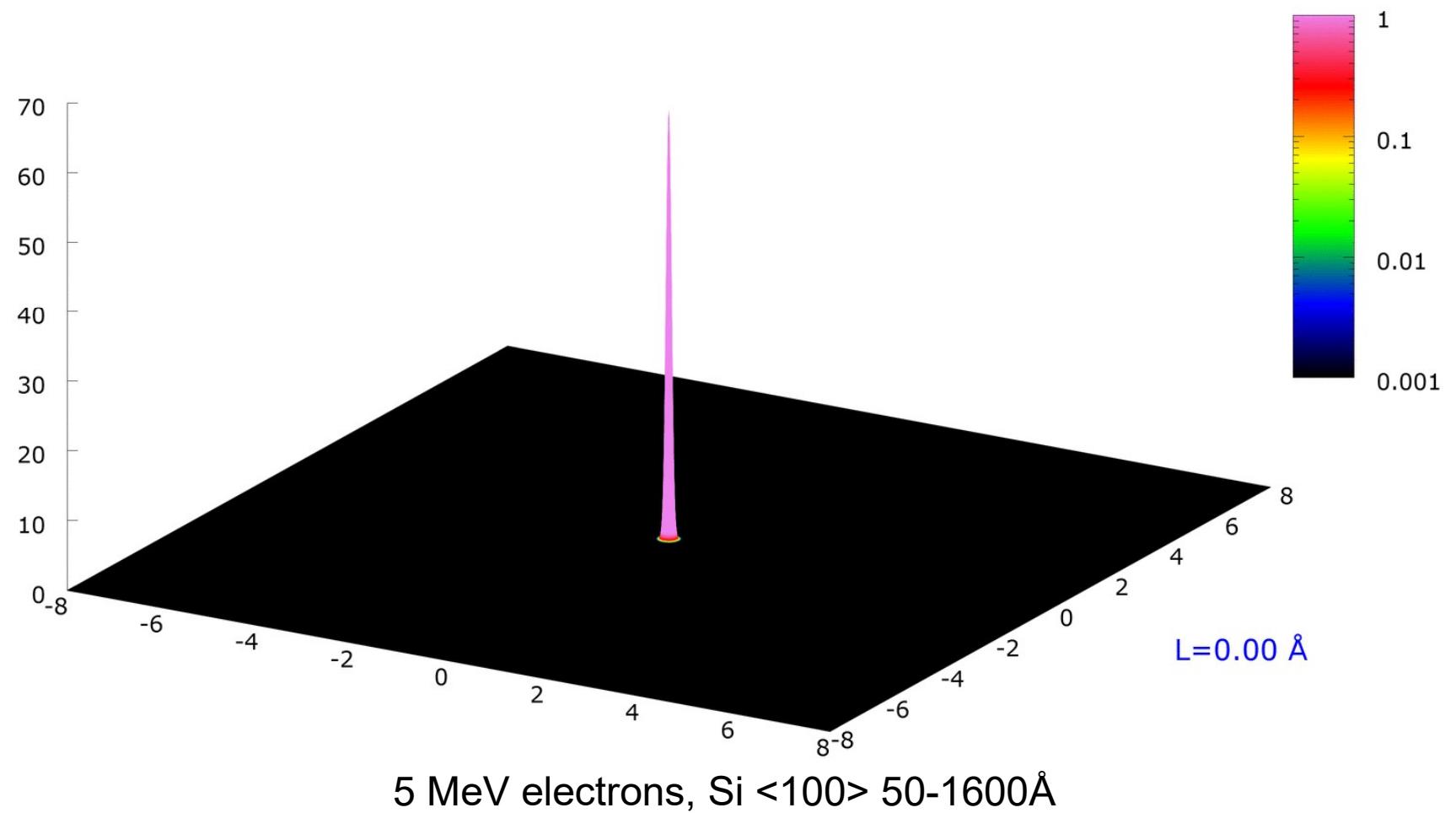


Quantum angular distributions of electrons in ultrathin Si <100> crystal



5 MeV electrons, Si <100> 50-1600Å

Quantum angular distributions of electrons in ultrathin Si <100> crystal



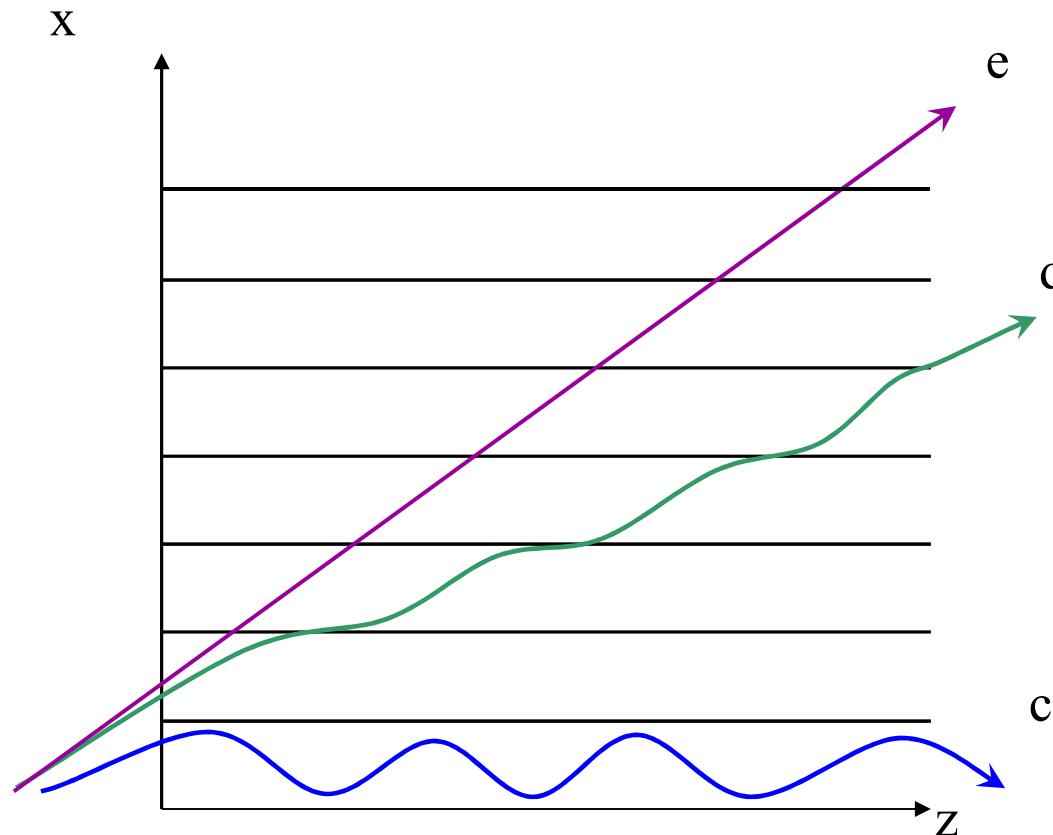
Conclusions

- Scattering in the fields of complex configuration
- Geometrical optics method
- Operator method
- Quantum and classical effects in scattering
- Scattering in transitional range of crystal thicknesses (from absence to presence of channeling)
- Analogue of the Ramsauer-Townsend effect for high energy particles
- ----- FUTURE -----
- How the quantum levels and zones do appear at regular motion and dynamical chaos?
- Coherent and incoherent scattering
- Bremsstrahlung and e^+e^- pair production in the geometrical optics approx.
-
- ...

Beam deflection by bent crystals

N. Shul'ga, I.Kirillin, V. Truten'

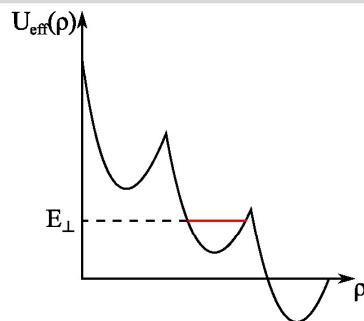
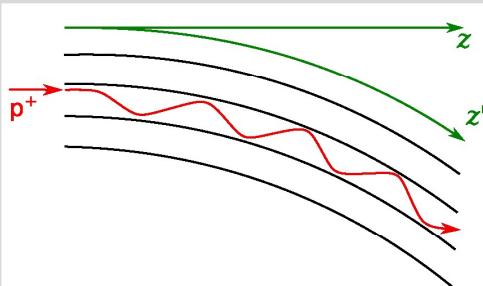
Mechanisms of Charged Particles Motion near Crystal Planes



Phenomena of channeling (J. Lindhard, 1965) and above-barrier motion
in crystal (A. Akhiezer, N. Shul'ga, 1978)

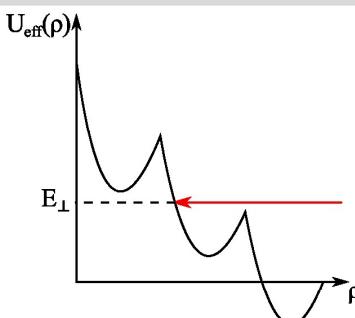
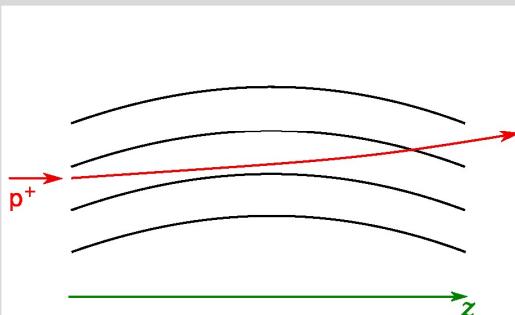
MECHANISMS OF HIGH-ENERGY CHARGED PARTICLE DEFLECTION BY BENT CRYSTALS

Planar channeling in bent crystal (*E.N. Tsyganov, 1976*)



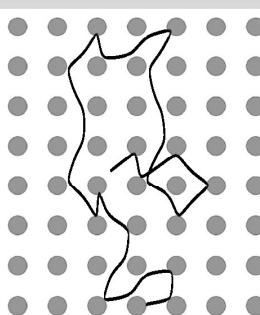
1979 – IHEP (Russia)
1980 – CERN

Volume reflection (*A.M. Taratin, S.A. Vorobiev, 1987*)



2006 – IHEP (Russia)
2006 – PNPI (Russia)
2007 – CERN

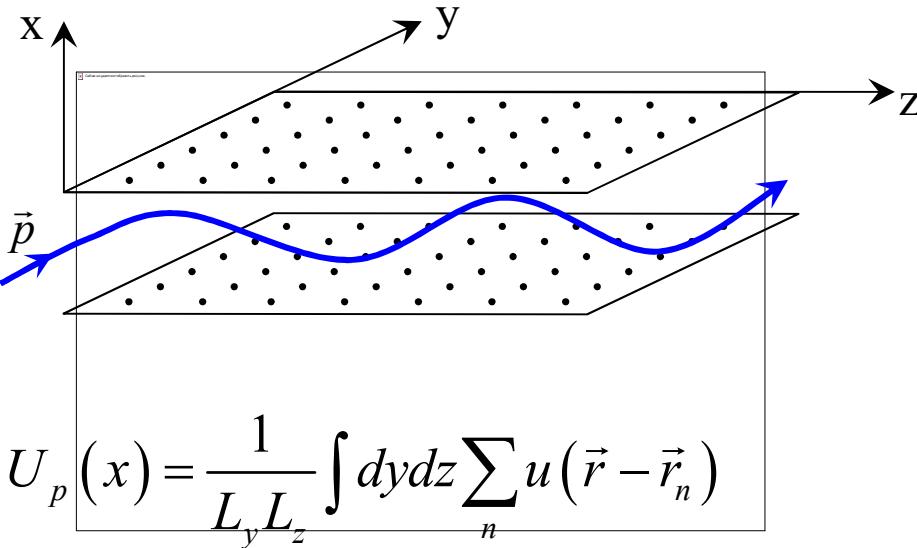
Stochastic deflection mechanism (*A.A. Greenenko, N.F. Shul'ga, 1991*)



2008 – CERN, protons
2009 – CERN, π^- -mesons

Planar Channeling (Regular Motion)

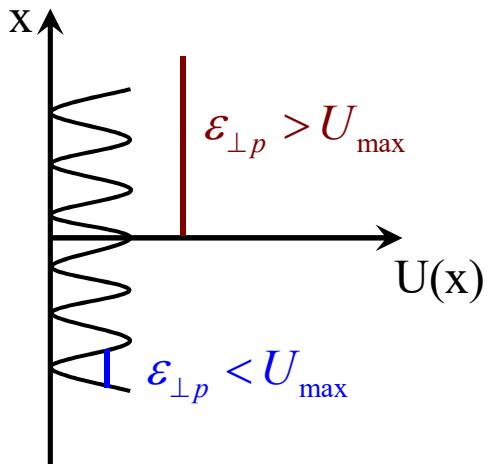
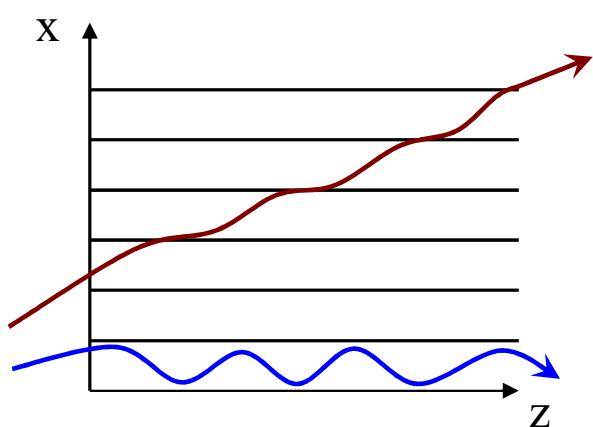
J. Lindhard (1965)



$$p_z = \text{const} \approx p$$

$$\ddot{x} = -\frac{1}{E} \frac{\partial}{\partial x} U_p(x)$$

$$\varepsilon_{\perp p} = \frac{E \dot{x}^2}{2} + U_p(x)$$



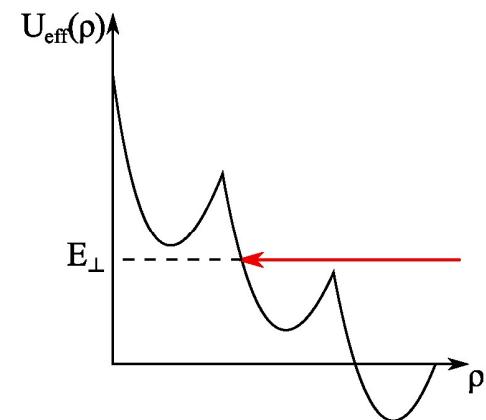
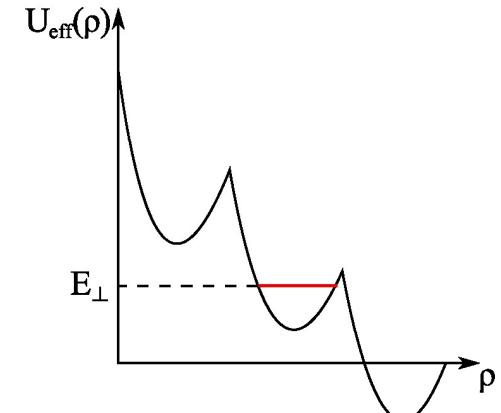
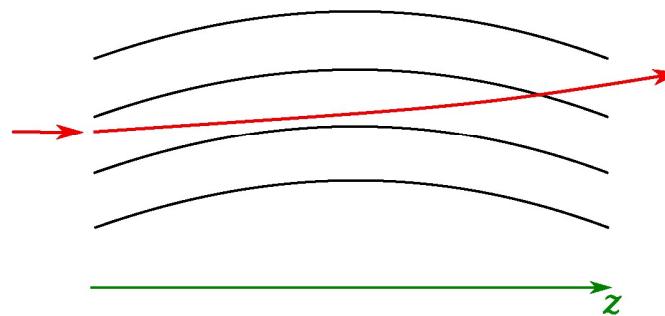
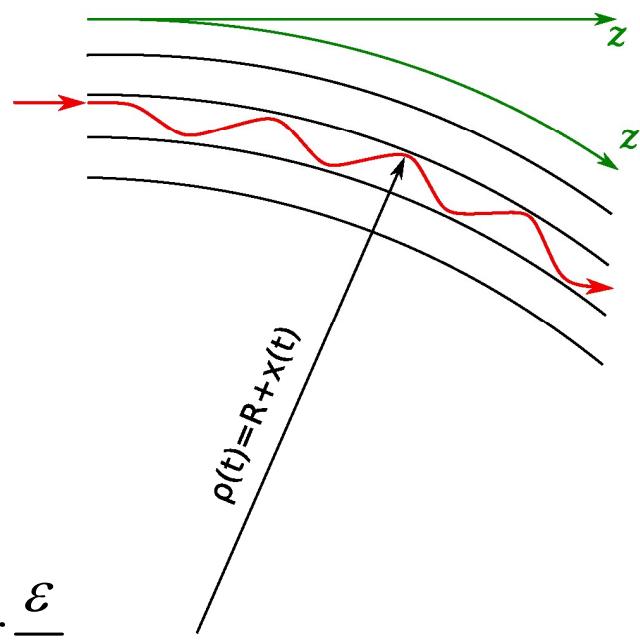
$$\varepsilon_{\perp p} = U_{\max} = \frac{E \theta_p^2}{2}$$

$$\theta_p = \sqrt{\frac{2U_{\max}}{E}}$$

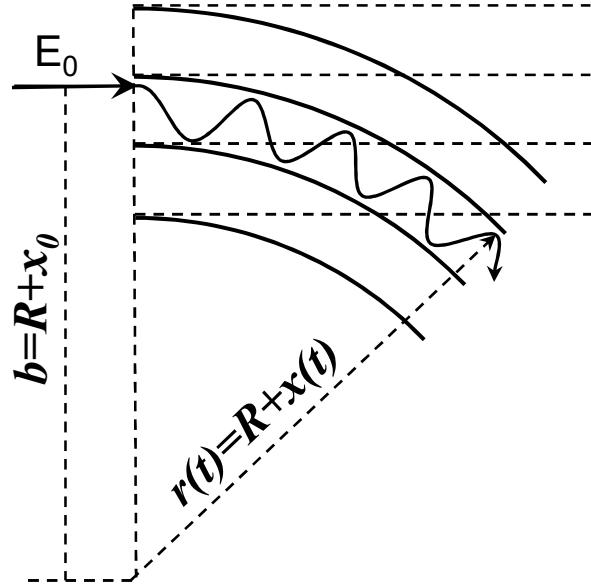
Planar Channeling and Above-Barrier Scattering in Bent Crystal

$$\ddot{x} = -\frac{1}{\varepsilon} \frac{\partial}{\partial x} U_{eff}(x)$$

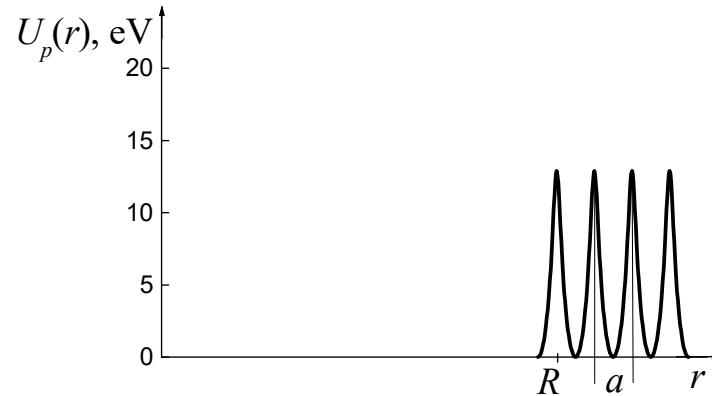
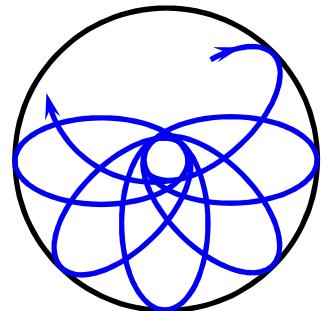
$$U_{eff}(x) = U(x) - x \frac{\varepsilon}{R}$$



The Motion of Relativistic Particle in Central Field of Bent Crystal Planes



Finite motion, precession



$$\frac{d^2x}{dt^2} = -\frac{1}{E} \frac{\partial}{\partial x} U_{eff}(x)$$

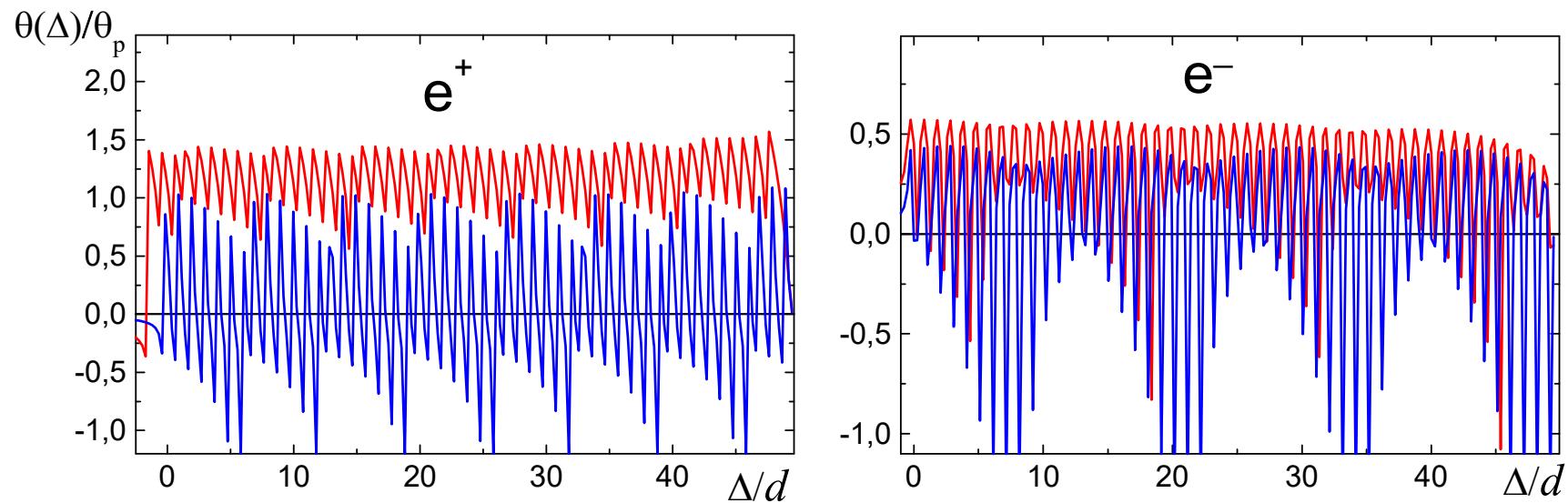
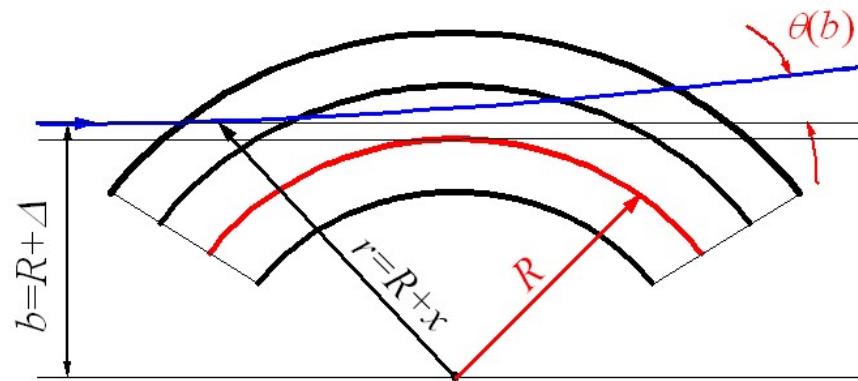
$$U_{eff}(x) = U_p(x) - x \frac{E}{R}$$

A. Akhiezer, N.Shul'ga, A.Greenenko et al., Sov.Phys. Usp. 1995

J. Ellison, Nucl. Phys. B 206 (1982) 205

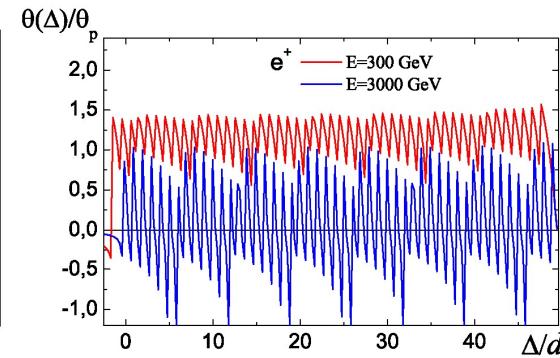
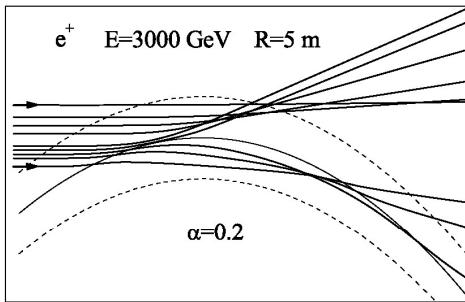
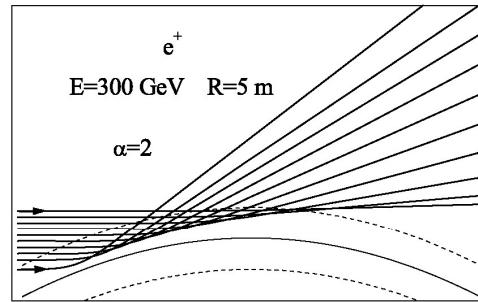
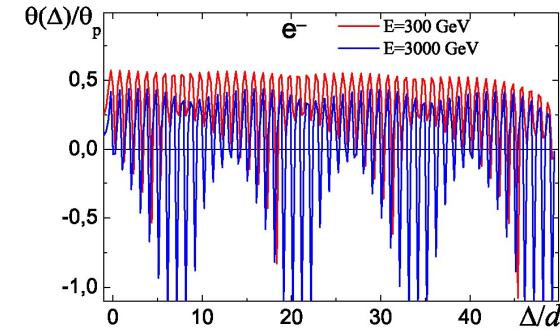
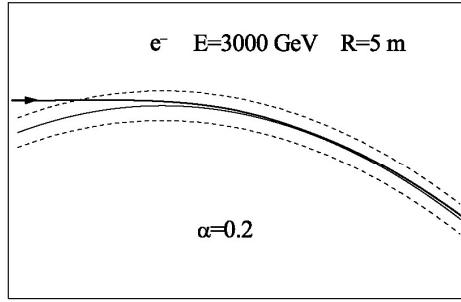
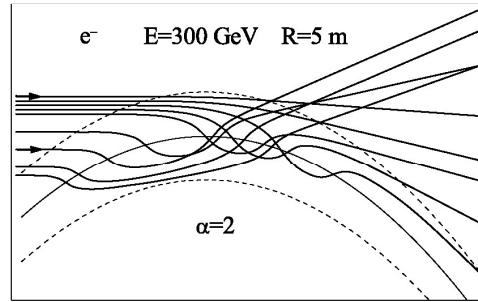
Deflection Functions for Beam Reflection in Crystal

N.F. Shul'ga, V.I. Truten', V.V. Boyko, 2009



— $E = 300 \text{ GeV}$ $\alpha = 2$.
— $E = 3000 \text{ GeV}$ $\alpha = 0.2$

CHARGED PARTICLE REFLECTION FROM BENT CRYSTAL ATOMIC PLANES



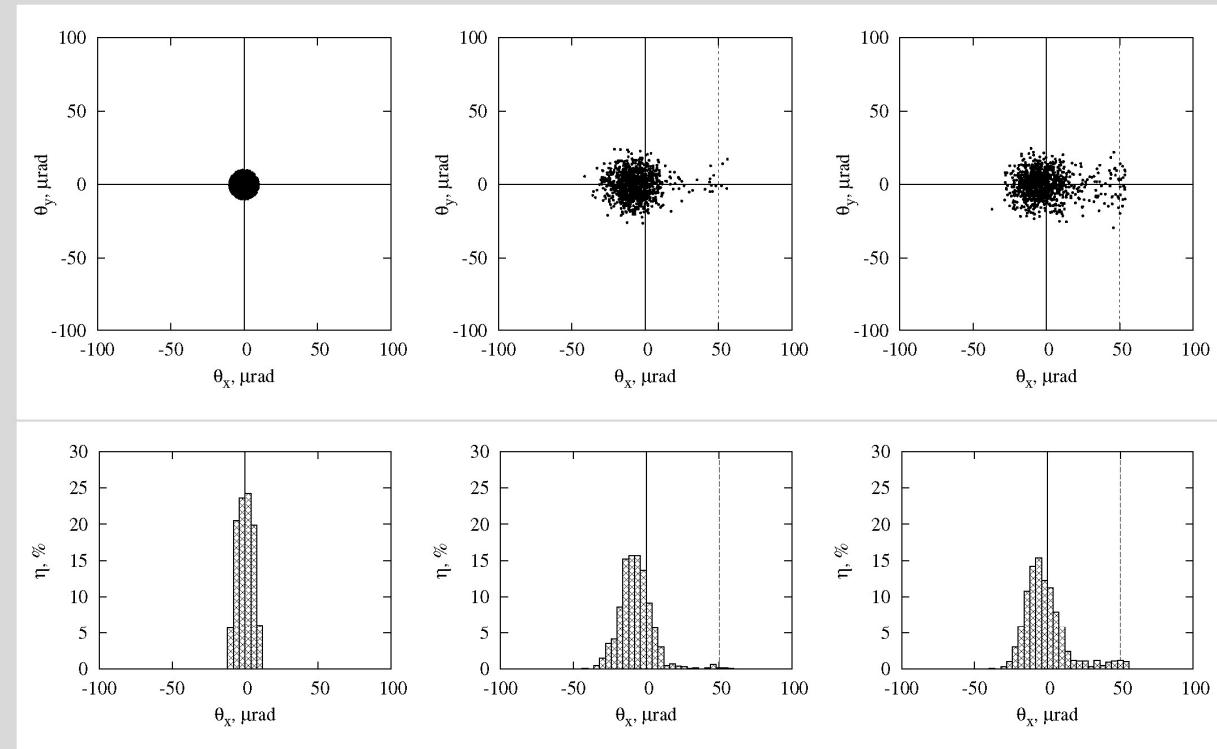
SIMULATION RESULTS

Charged particle reflection from bent crystal atomic planes

particle beam before
entering the crystal

protons

π^- -mesons

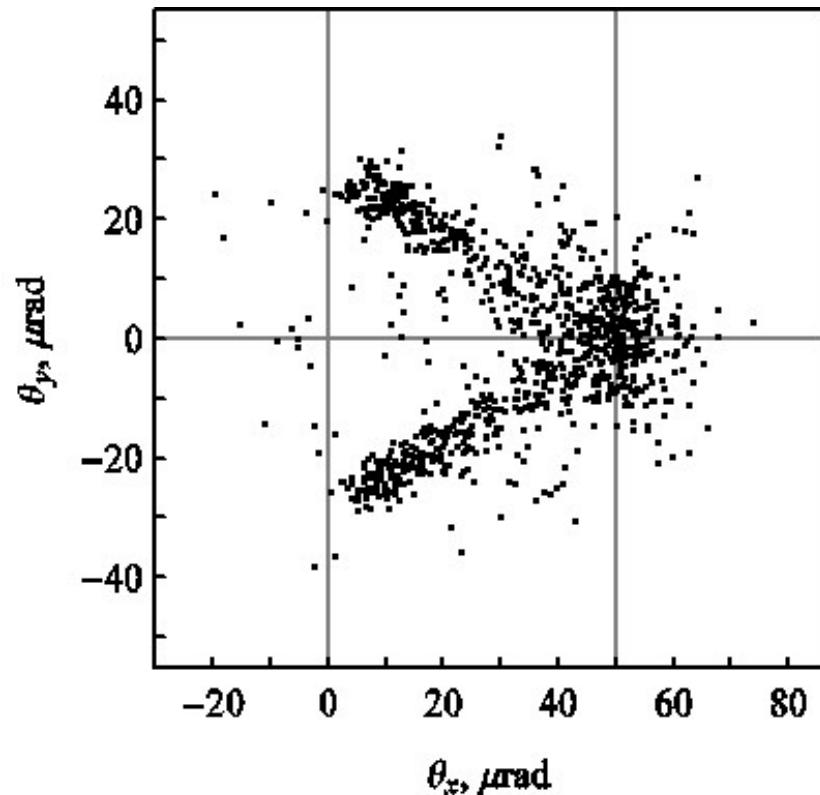


Beam center before entering the crystal had angular coordinates (50,500) μ rad with
relative to the $\langle 110 \rangle$ crystal axis
(500μ rad $\approx 50 \theta_c$)

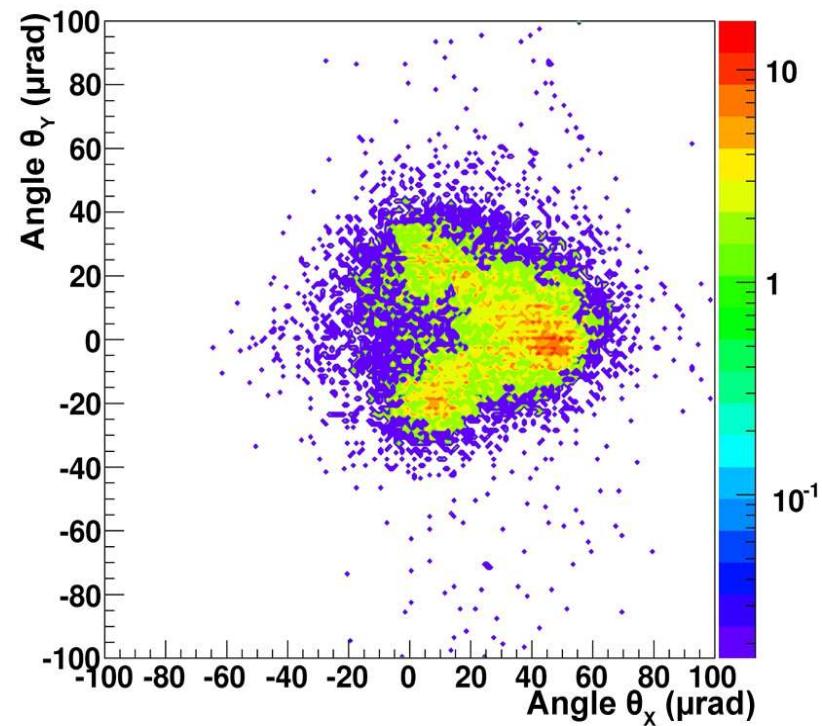
$E=1$ TeV, $L=2$ cm, $R=200$ m

Collaboration CERN UA9

*Angular distribution of 400 GeV protons after passing
2 mm of bent Si crystal with $R=40$ m*



Simulation results

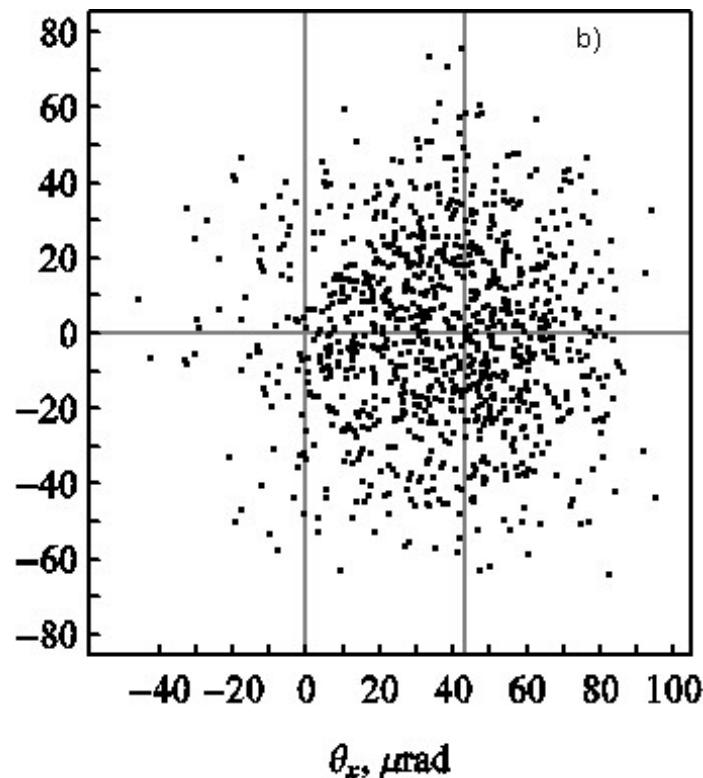


CERN experiment

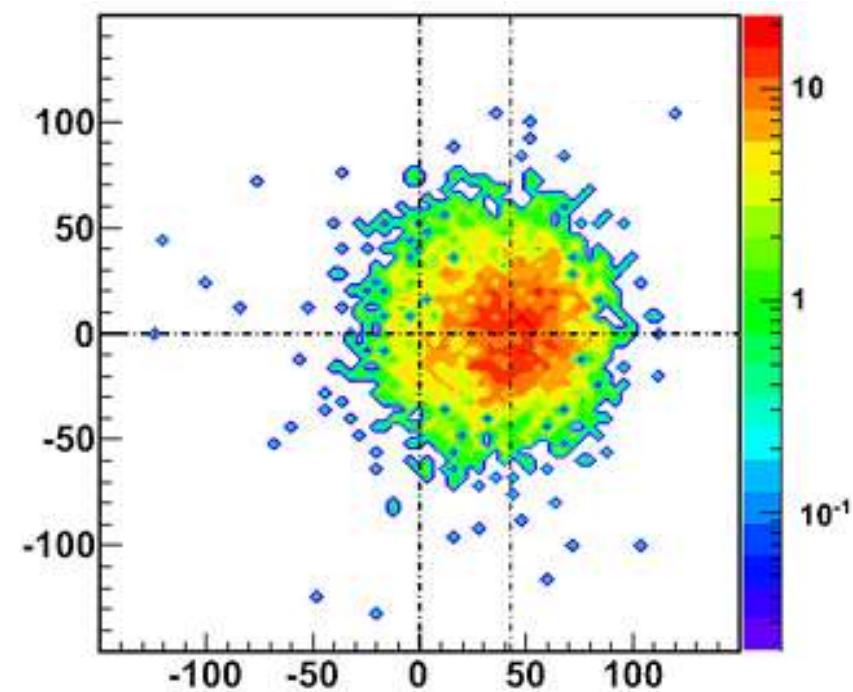
*W. Scandale et al. Phys. Rev. Lett.
101 (2008), 164801*

Collaboration CERN UA9

*Angular distribution of 150 GeV π^- -mesons after passing
1.172 mm of bent Si crystal with $R=40$ m*



Simulation results



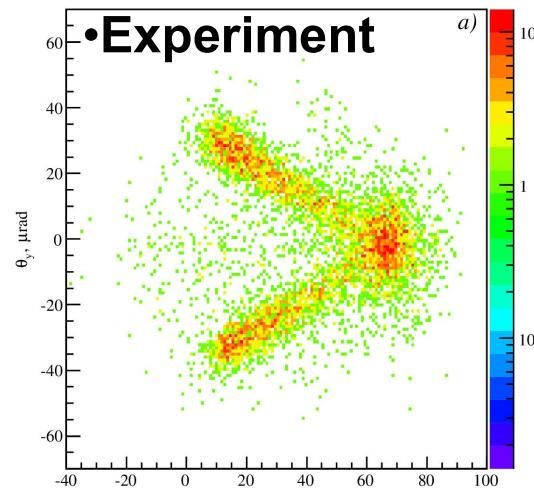
CERN experiment

*W. Scandale et al. Physics Letters B
680 (2009) 301-304*

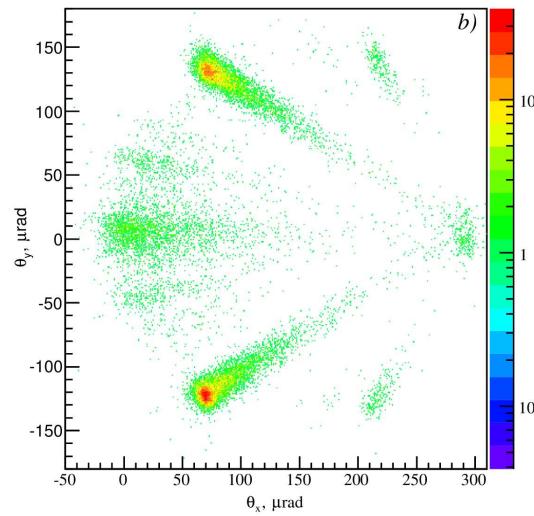
Collaboration CERN UA9

Proton beam splitting by a bent crystal at SPS CERN (2010-2016)

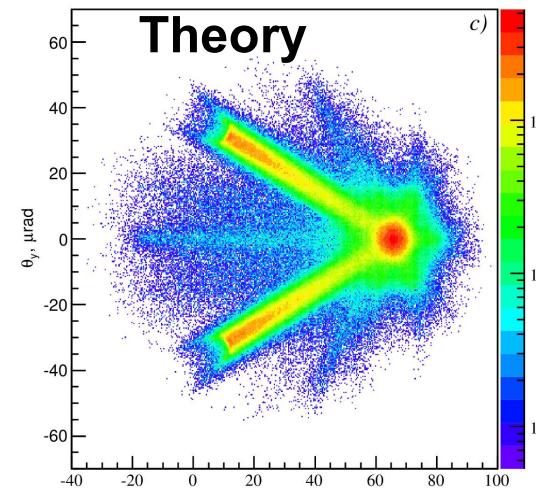
• $R = 30.3 \text{ m}$



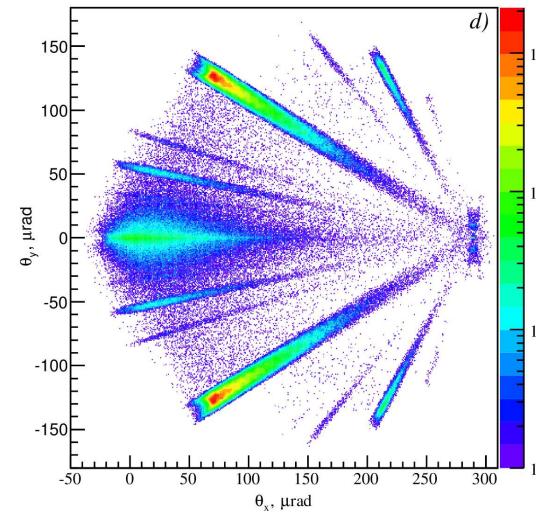
• $R = 6.9 \text{ m}$



• $E = 400 \text{ GeV}$

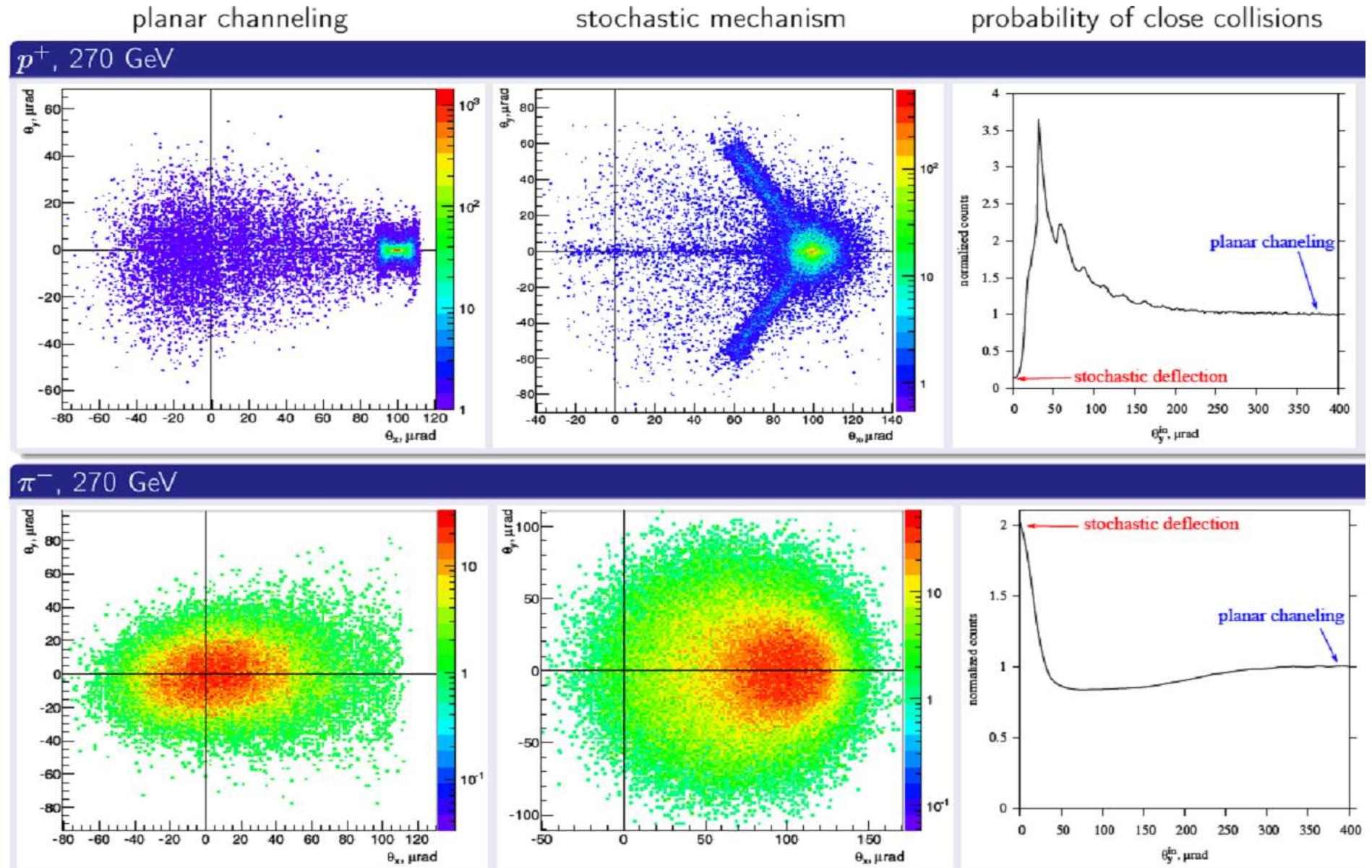


• **Si, <111>**



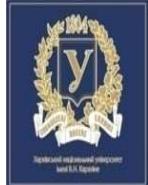
• V. Guidi, I. V. Kirillin, N. F. Shul'ga et al. // Eur. Phys. J. C (2016) 76:80.

Probability of close collisions in bent crystal



Proposal by KIPT and LAL for CERN experiment (2015)

Yu.A. Chesnokov, I.V. Kirillin, W. Scandale et al. // Phys. Lett. B. – 2014. – Vol. 731. – P. 118–121.



Proposal for CERN UA9 (J. High Energ. Phys. (2017) 2017: 120)

Feasibility of measurement of the magnetic moments of the charm baryons at the LHC using bent crystals

O.A. Bezshyyko,¹ L. Burmistrov,² A.S. Fomin,^{2,3,4,*} S.P. Fomin,^{3,4} I.V. Kirillin,^{3,4} A.Yu. Korchin,^{3,4,†}
L. Massacrier,⁵ A. Natochii,^{1,2} P. Robbe,² W. Scandale,^{2,6,7} N.F. Shul'ga,^{3,4} and A. Stocchi^{2,‡}

¹*Taras Shevchenko National University of Kyiv, 01601 Kyiv, Ukraine*

²*LAL (Laboratoire de l'Accélérateur Linéaire), Université Paris-Sud/IN2P3, Orsay, France*

³*NSC Kharkiv Institute of Physics and Technology, 61108 Kharkiv, Ukraine*

⁴*V.N. Karazin Kharkiv National University, 61022 Kharkiv, Ukraine*

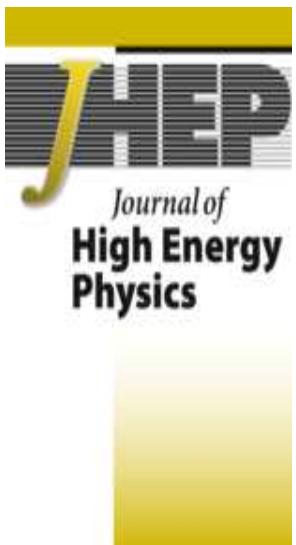
⁵*IPNO (Institut de Physique Nucléaire), Université Paris-Sud/IN2P3, Orsay, France*

⁶*CERN, European Organization for Nuclear Research, CH-1211 Geneva 23, Switzerland*

⁷*INFN Sezione di Roma, Piazzale Aldo Moro 2, 00185 Rome, Italy*

(Dated: April 28, 2017)

In this paper we revisit the idea of measuring the magnetic dipole moments of the charm baryons and in particular of Λ_c^+ by studying the spin precession induced by the strong effective magnetic field inside the channels of a bent crystal. We present a detailed sensitivity study showing the feasibility of such an experiment at the LHC in the coming years.



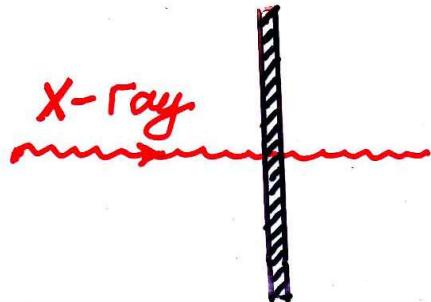
Electromagnetic processes with high energy “half-bare” electrons

(Experiments in 2005-2010 and proposals for CERN NA63)

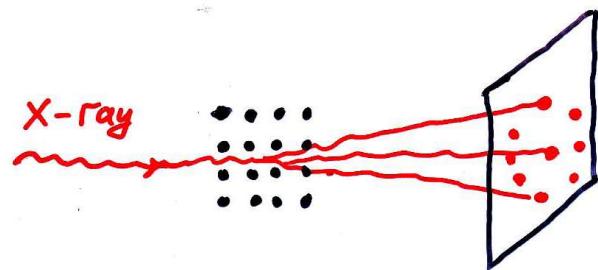
- LPM and TSF effects
- TSF effect in thin crystals
- Ionization energy losses
- Transition radiation
- Pair production
- Chudakov effect
- Electromagnetic showers
-
-
-

THANK YOU FOR YOUR ATTENTION!

Channeling (history)

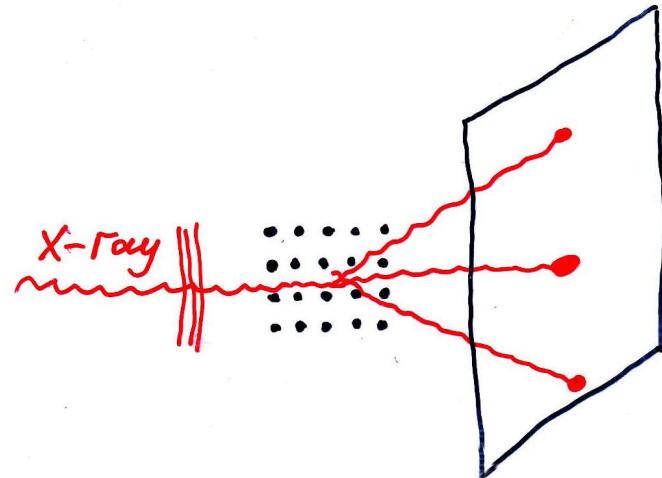


W. Röntgen (1896)
Penetration through matter
Nobel Prize 1901

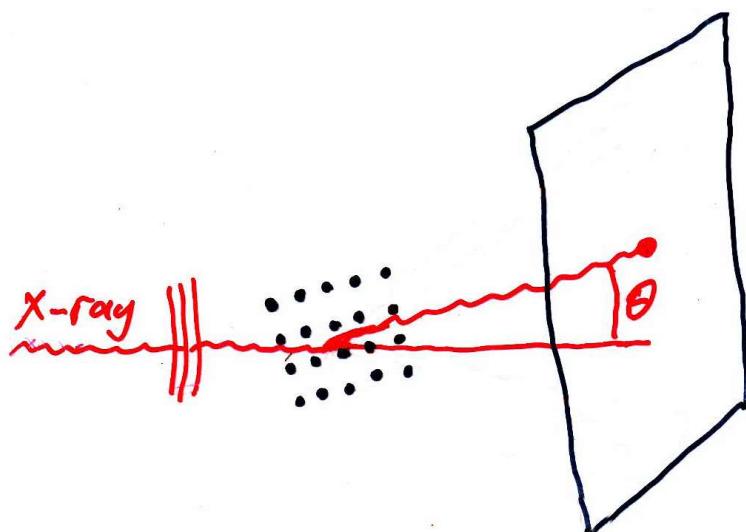


M.Laue, W.Fridrich, P.Knipping (1912)
Diffraction in crystal (Laue spots)
Nobel Prize 1914

Channeling (history)

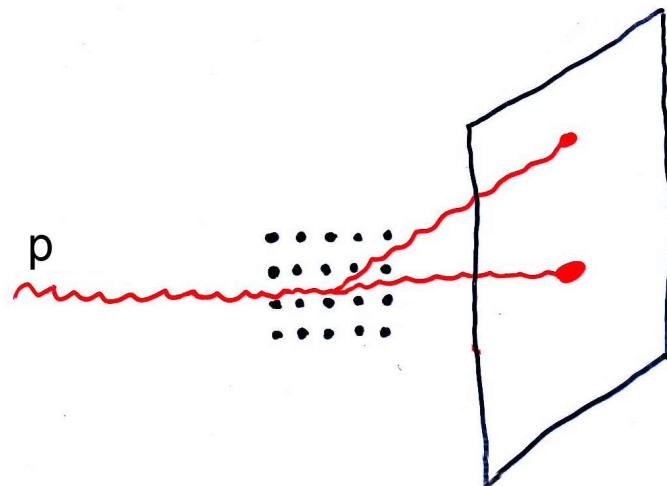


H. Bragg (1912)
The nature of Laue spots?
X-Rays are particles
Open channels!!!

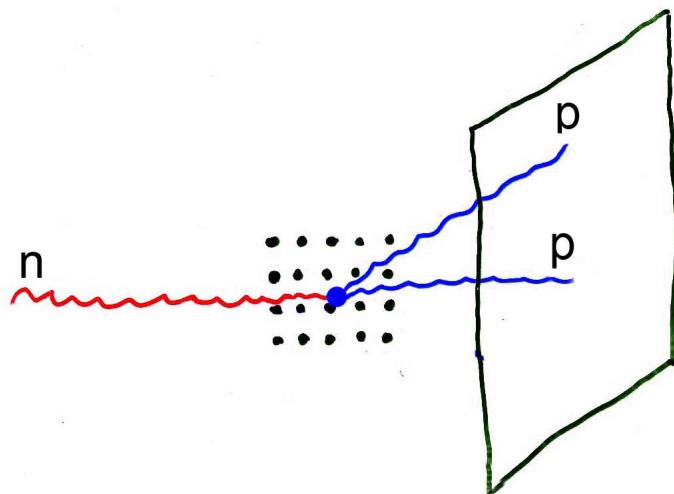


H. Bragg, L. Bragg (1912)
Proposal of experiment
They searched $\theta_L = \theta$!
They found $\theta_L = 2\theta$!!! (Bragg's reflection)
Nobel Prize 1915

Channeling (history)



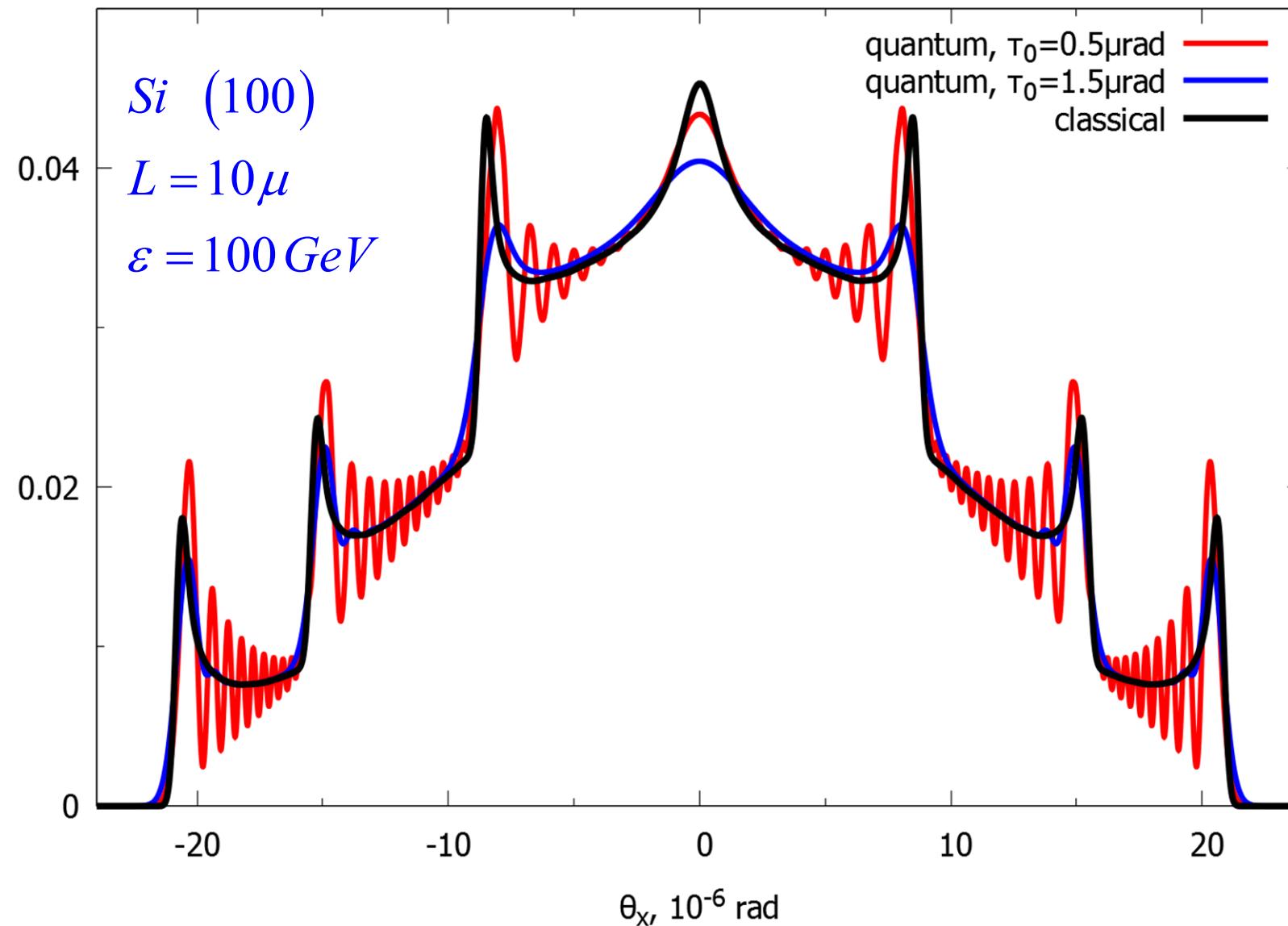
J. Stark (1912)
Hypothesis
For charged particles – open channels



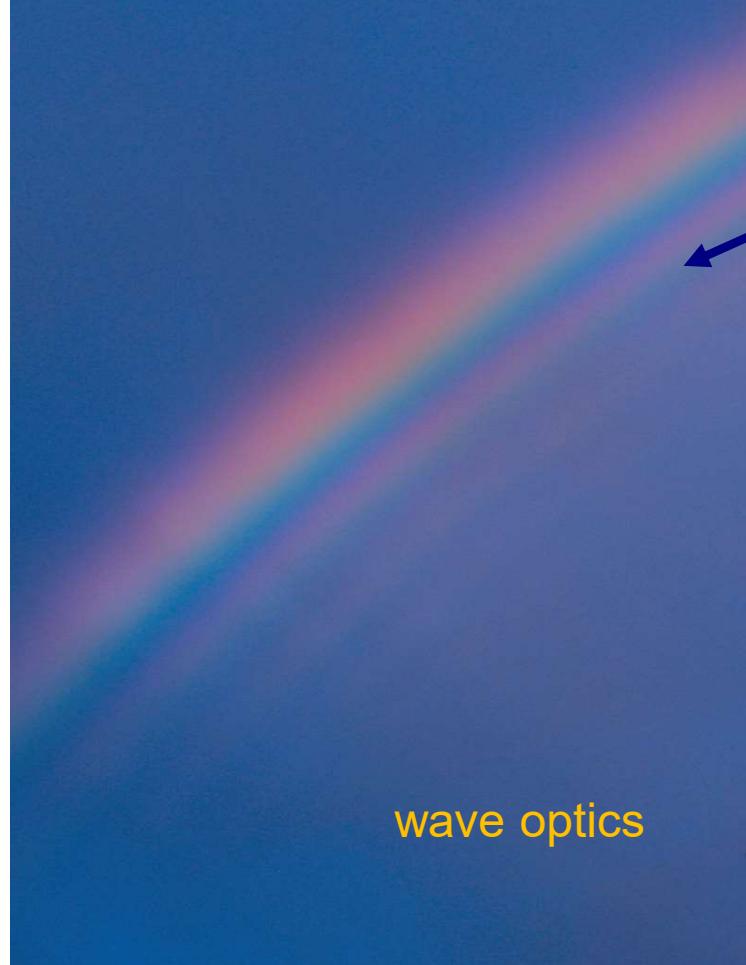
M. Robinson, O. Oen (1961)
Numerical simulation of nuclear
reactions in crystal

J. Lindhard (1965)
Theory of open channels (channeling)

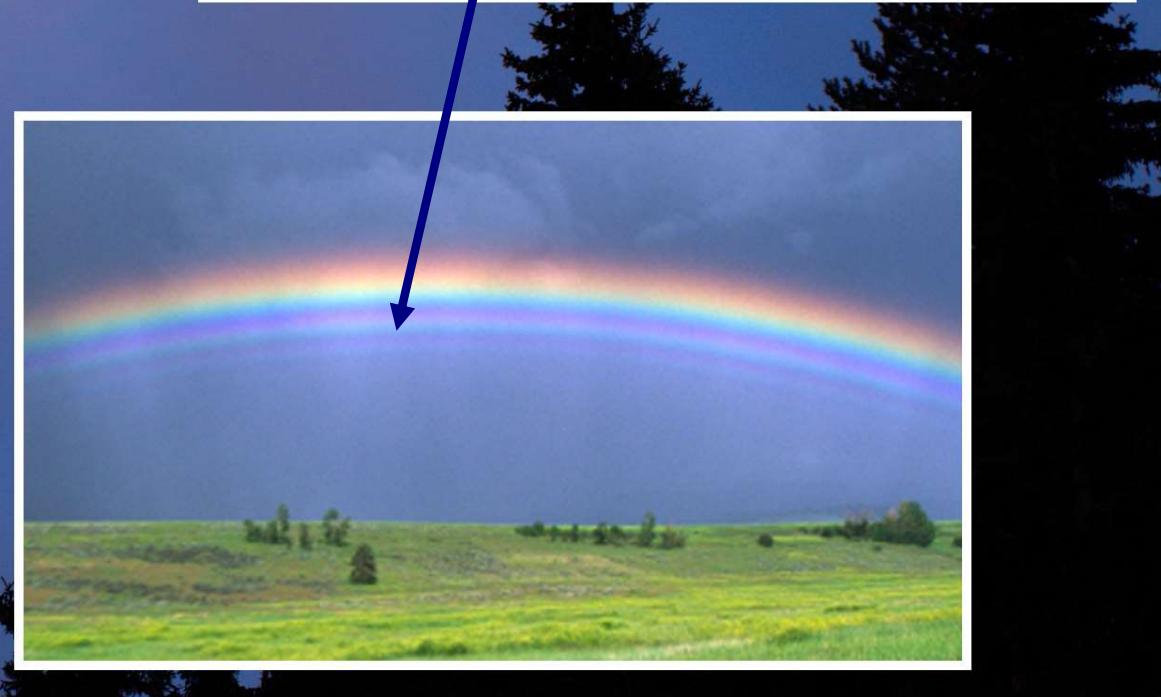
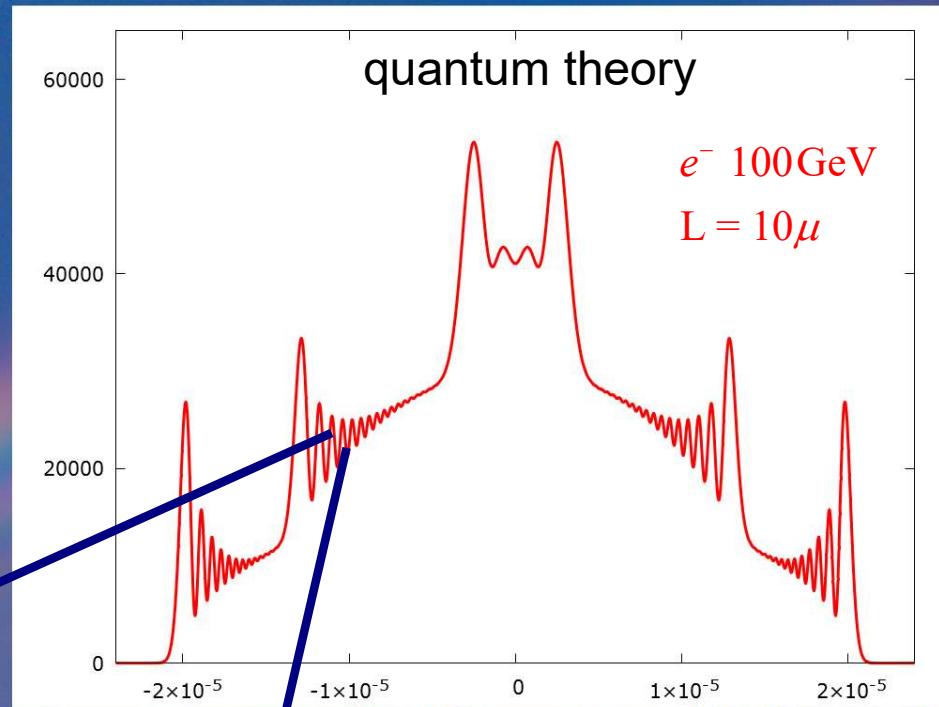
Rainbow scattering in the field of ultrathin Si (110) crystal planes



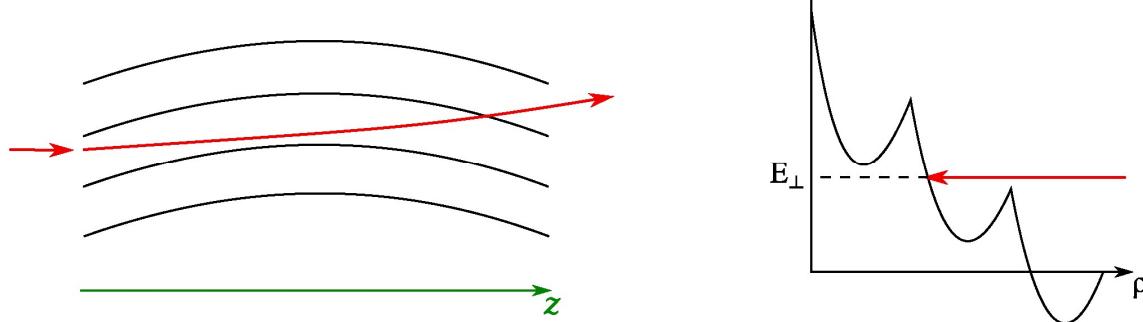
Rainbow scattering in the field of in the field of ultrathin Si (110) crystal planes



wave optics



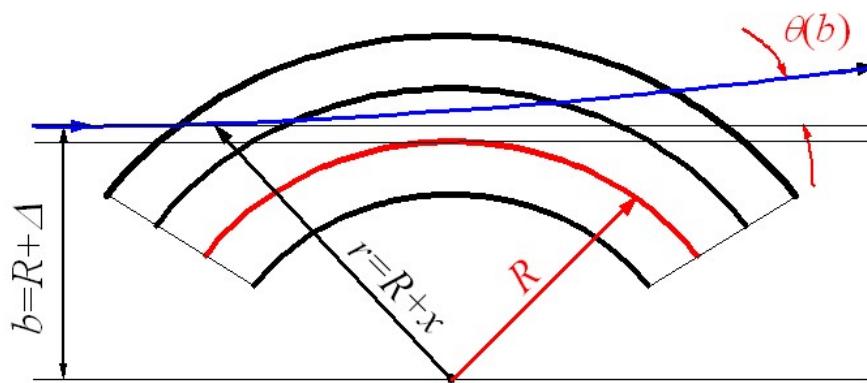
CHARGED PARTICLE REFLECTION FROM BENT CRYSTAL ATOMIC PLANES



Particle reflection in the field of bent atomic planes can be considered as particle scattering in a cylindrically symmetric field.

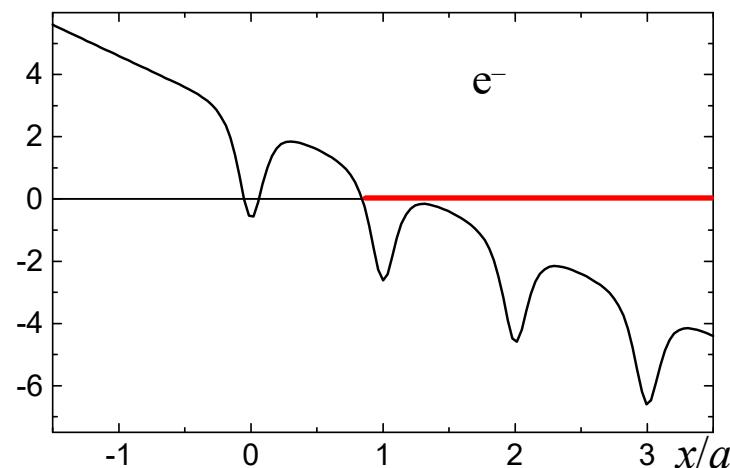
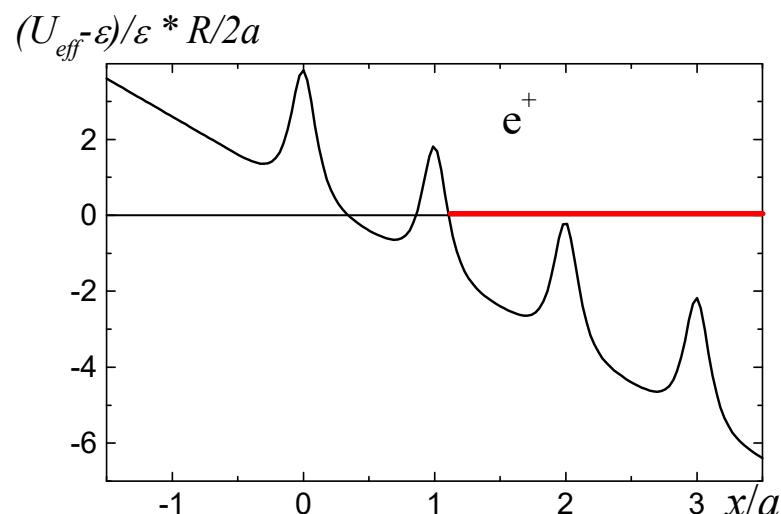
$$\theta(b) = \pi - 2Mc \int_{\rho_0}^{\infty} \frac{d\rho}{\rho^2 \sqrt{(E - U(\rho))^2 - M^2c^2/\rho^2 - m^2c^4}},$$

Potential for Beam Reflection by Bent Crystal Planes



$$\vartheta(b) = \pi - 2b\nu\sqrt{E} \int_{r_0}^{\infty} \frac{dr/r^2}{\sqrt{E - U_{eff}(r, b)}}$$

$$U_{eff}(r, \Delta) = E + 2U(x) + 2E(\Delta - x)/R$$

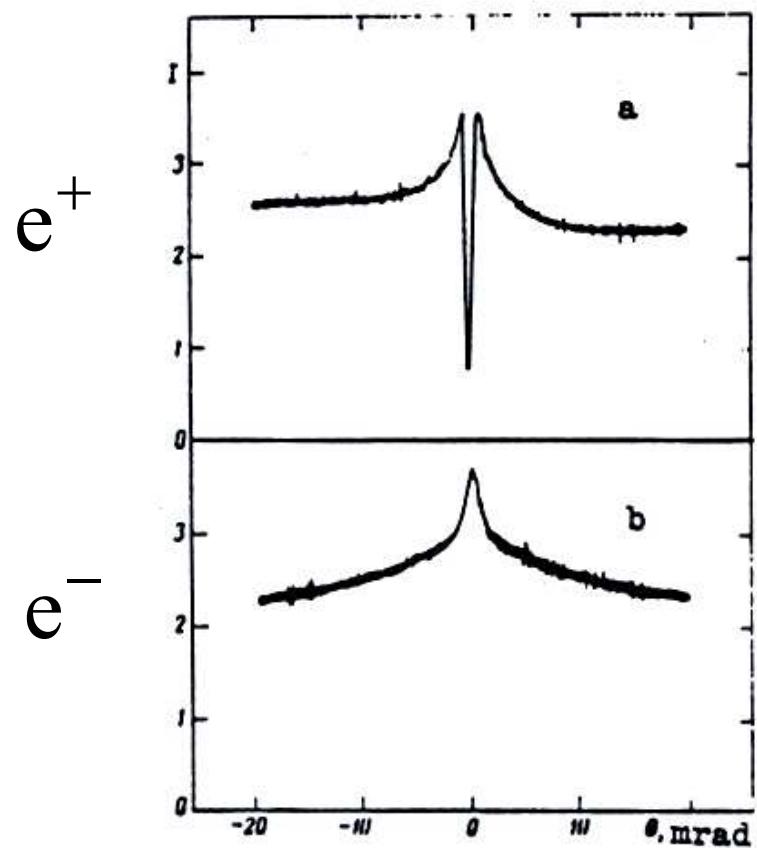


Condition for bending: $\alpha = \frac{4U_0}{E} \frac{R}{d} \gg 1$

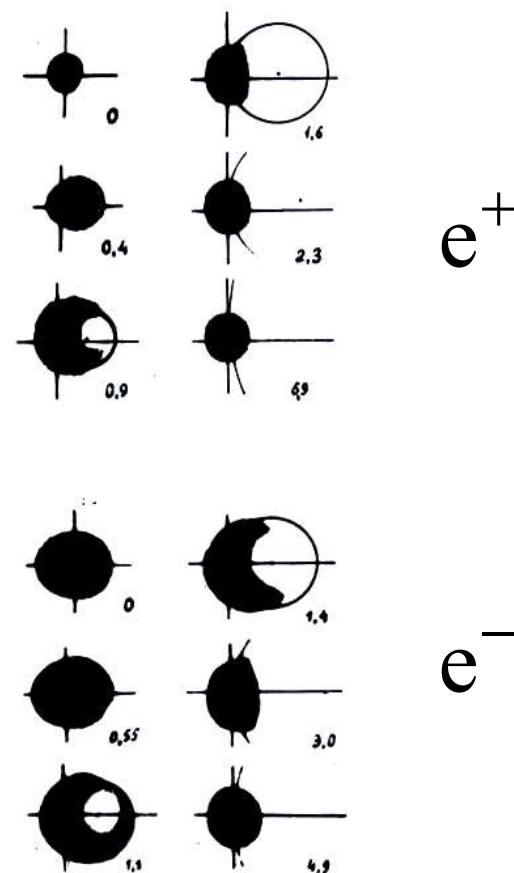
Channeling and coherent bremsstrahlung and scattering for 1 GeV positrons and electrons

(Ukraine, Kharkov, 1972)

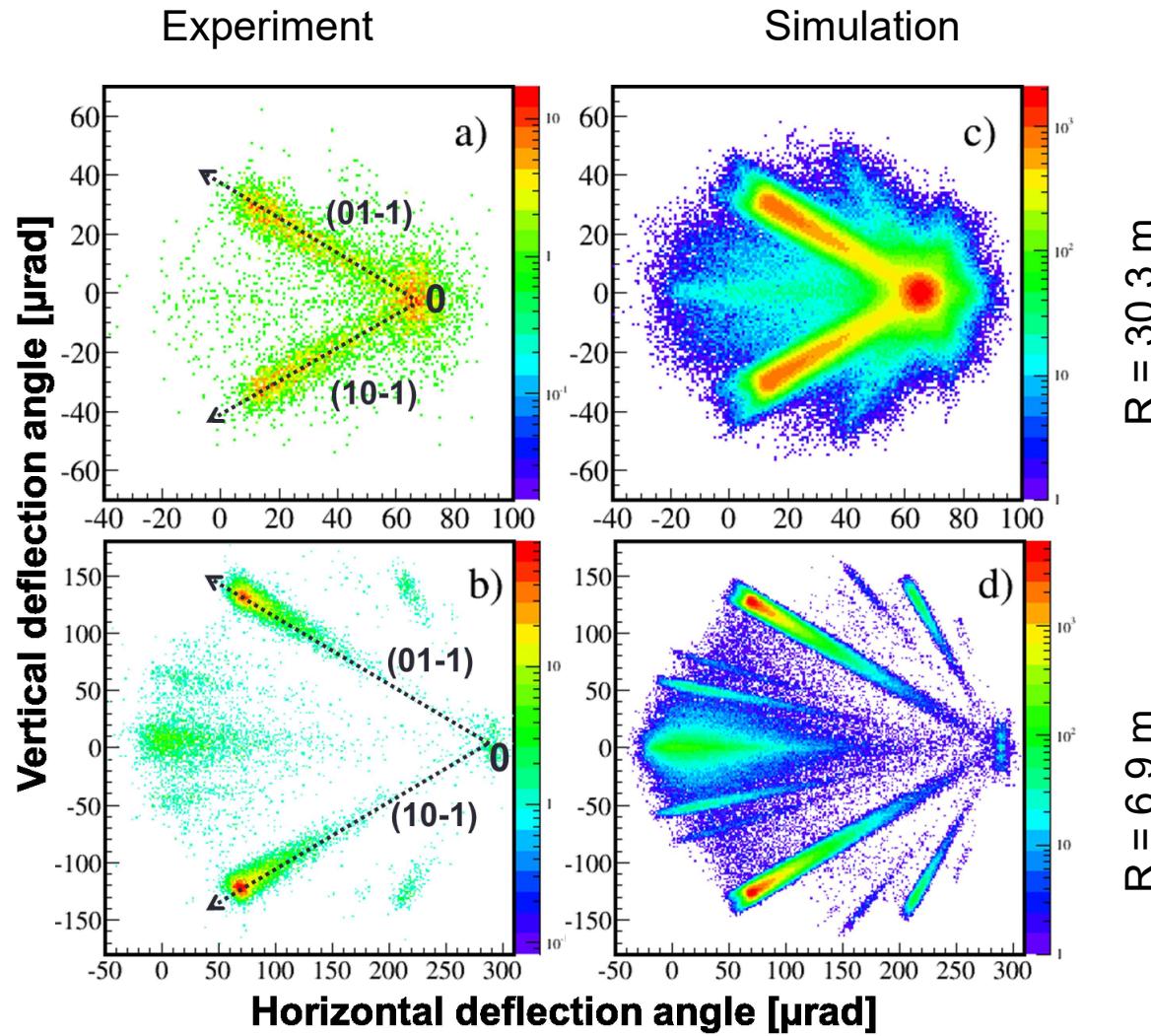
Orientation dependence
of the radiation intensity



Angular distribution
of scattering



Stochastic deflection and beam splitting



V. Guidi, I. V. Kirillin, N. F. Shul'ga et al. // Eur. Phys. J. C (2016) 76:80.