Magnetic and electric dipole moments of short-lived particles and proposal for their measurement at CERN

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Outline of the talk

- Stable and unstable particles in the Standard model
- Magnetic and electric dipole moments: classical electrodynamics, quantum mechanics: Schrodinger and Dirac equations
- Spin magnetic dipole moment: problem "g-2" for electron, muon, quarks
- Electric dipole moment (EDM): violation of T- and CP-invariance. Baryon asymmetry in the Universe
- Magnetic dipole moment (MDM) of charmed baryons
- Spin precession in external magnetic and electric fields
- Feasibility of measuring MDM and EDM of short-lived particles using bent crystals
- Proposals for experiments at CERN

Stable and unstable particles

- Stable particles: proton $p(uud)$ (antiproton $\bar{p}(\bar{u}\bar{u}\bar{d})$),
  electron e^- (positron e^+), photon,
- Almost stable: neutron $n(udd)$ (antineutron $\bar{n}(\bar{u}\bar{d}\bar{d})$, lifetime $\tau \approx 15$ min,
- Unstable particles: muon μ^- (antimuon μ^+), lifetime $\tau = 2.2 \times 10^{-6}$ s,
  tau lepton τ^- (antitau τ^+), lifetime $\tau = 2.9 \times 10^{-13}$ s,
- Light "strange" baryons (and antibaryons): $\Lambda(uds)$, lifetime $\tau = 2.6 \times 10^{-10}$ s,
  $\Sigma^+(uus)$, lifetime $\tau = 0.8 \times 10^{-10}$ s, etc.,
- Heavy "charmed" baryons: $\Lambda_c^+(udc)$, lifetime $\tau = 2 \times 10^{-13}$ s,
  "strange-charmed" baryons $\Xi_c^+(usc)$, lifetime $\tau = 4.4 \times 10^{-13}$ s, etc. ,
  "Bottom" baryons: $\Lambda_b^0(udb)$, lifetime $\tau = 1.47 \times 10^{-12}$ s, etc.,
- Baryons resonances (decaying due to the strong interaction): e.g., Δ, lifetime $\tau \sim 10^{-23}$ s, and many others.

Electric and magnetic dipole moments in classical electrodynamics

L. Landau, E. Lifshitz, "Field Theory", J.D. Jackson "Classical electrodynamics", 1999

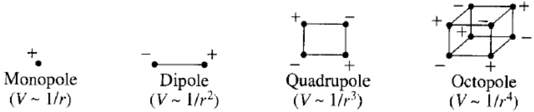
Electric dipole moment (EDM) of a system of particles.

$$\phi(\vec{R}) = \sum_{a=1}^{N} \frac{e_a}{|\vec{R} - \vec{r}_a|} \approx \frac{Q}{R} - \frac{\vec{d}\vec{n}}{R^2} + \frac{D_{ij} n_i n_j}{2R^3} + \cdots$$

where $\vec{n} \equiv \vec{R}/R$, $Q \equiv \sum_a e_a$ - total electric charge, and $R \gg |\vec{r}_a|$ for all a.

$$\vec{d} = \sum_a e_a \vec{r}_a = \int \vec{r} \rho(\vec{r})\, d^3 r \; - \; \textit{dipole moment},$$

$$D^{ij} = \sum_a e_a (3x_a^i x_a^j - r_a^2 \delta^{ij}) = \int (3x^i x^j - r^2 \delta^{ij}) \rho(\vec{r})\, d^3 r \; - \; \textit{quadrupole moment}$$



System in external electric field

Now put the system in **external field** with the potential $\phi(\vec{r})_{ext}$:

$$U = \sum_a e_a \phi(\vec{r}_a)_{ext} \approx Q\phi(0)_{ext} - \vec{d}\vec{E}_{ext} + \frac{1}{6} Q^{ij} \nabla^i E^j_{ext} + \cdots$$

There is also a torque ("вращающий момент") which acts on the system

$$\vec{K} = \sum_a \vec{r}_a \times \vec{F}_a = \sum_a \vec{r}_a \times e_a \vec{E}_{ext} = \vec{d} \times \vec{E}_{ext}$$

Therefore we see that the potential energy of the dipole and the torque are related

$$U = -\vec{d}\vec{E}_{ext} = -d\, E_{ext} \cos\alpha, \qquad |\vec{K}| = d\, E_{ext} \sin\alpha = \frac{\partial}{\partial\alpha} U,$$

so that at the angle $\alpha = 0$ the torque vanishes and the potential energy takes minimal value $-E_{ext} d$. This means that the electric dipole tends to be directed along the electric field $\vec{E}_{ext}$.

Magnetic dipole moment (MDM)

For a system of charges moving in the finite region of space one has vector potential

$$\vec{A} = \frac{1}{c} \sum_a \frac{e_a \vec{v}_a}{|\vec{R} - \vec{r}_a|} \approx -\frac{\vec{\mu}\vec{n}}{R^2} + \mathcal{O}(\frac{1}{R^3})$$

with MDM of the system

$$\vec{\mu} = \frac{1}{2c} \sum_a e_a (\vec{r}_a \times \vec{v}_a) = \sum_a \frac{e_a}{2m_a\, c} \vec{L}_a$$

where $\vec{L}_a = \vec{r}_a \times \vec{p}_a$ is the orbital moment of particle a.

Magnetic dipole in external magnetic field

Suppose we put the system in **external constant magnetic field** $\vec{B}_{ext}$. Vector potential is

$$\vec{A}(\vec{r})_{ext} = \frac{1}{2}(\vec{B}_{ext} \times \vec{r})$$

The potential energy of the system in external magnetic field is

$$U = -\frac{1}{c}\sum_a e_a \vec{v}_a \, \vec{A}(\vec{r}_a)_{ext} = -\vec{\mu}\, \vec{B}_{ext}$$

The force acting on the system $\vec{F} = -\vec{\nabla}(-\vec{\mu}\vec{B}_{ext}) = 0$, however the torque ("вращающий момент") is not zero

$$\vec{K} = \sum_a \vec{r}_a \times \vec{F}_a = \sum_a \vec{r}_a \times \frac{e_a}{c}(\vec{v}_a \times \vec{B}_{ext}) = \vec{\mu} \times \vec{B}_{ext}$$

We see that, like for the electric dipole, we have

$$U = -\mu\, B_{ext} \cos\beta, \qquad K = \mu\, B_{ext} \sin\beta$$

and the potential energy is minimal if $\beta = 0$, then $K = 0$ (no torque) and magnetic moment tends to be directed along the magnetic field, $\vec{\mu} \parallel \vec{B}_{ext}$.

Magnetic moments in quantum physics

First consider non-relativistic quantum mechanics based on **Schrodinger** equation.

E. Schrodinger

Hamiltonian of a particle in external electromagnetic field $A^\mu = (\phi, \vec{A})$ is

$$H = \frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A})^2 + e\phi = \frac{\vec{p}^2}{2m} + e\phi - \frac{e}{mc}\vec{A}\,\vec{p} + i\hbar\vec{\nabla}\vec{A} + \frac{e^2}{2mc^2}\vec{A}^2$$

If we have only constant magnetic field $\vec{B}$, then $\phi = 0$ and $\vec{A}(\vec{r}) = \frac{1}{2}(\vec{B} \times \vec{r})$. Then $\vec{\nabla}\vec{A} = 0$ and

$$H = \frac{\vec{p}^2}{2m} - \vec{\mu}\,\vec{B} + \mathcal{O}(e^2)$$

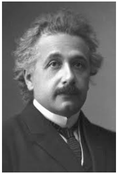
$$\vec{\mu} = \frac{e}{2m}(\vec{r} \times \vec{p}) = \frac{e}{2m}\vec{L},$$

where $\vec{L}$ is operator of orbital moment. Like in classical electrodynamics, MDM is determined by mechanical moment (orbital moment).

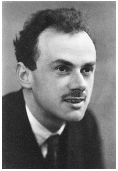
However the spin MDM does not appear in non-relativistic description.

Magnetic moments in quantum physics: Dirac equation

Unification of special relativity and quantum mechanics



A. Einstein



P.A.M. Dirac

The famous Dirac equation (linear in derivatives) for spin-1/2 fermion in external electromagnetic field $A^\mu = (A^0, \vec{A})$

$$i\frac{\partial}{\partial t}\psi = [\, c\vec{\alpha}\,(\vec{p} - \frac{e}{c}\vec{A}) + \beta mc^2 + eA^0\,]\,\psi,$$

for the 4-component spinor $\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$, and β and $\vec{\alpha}$ are 4×4 Dirac matrices.

Magnetic moments in quantum physics: spin and MDM

For non-relativistic velocities we need to reduce the Dirac eq-n to equation for the Pauli equation for the 2-component spinor $\varphi \quad (\varphi \gg \chi)$:

$$i\frac{\partial}{\partial t}\varphi = \left[\frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A})^2 + eA^0 - \frac{e\hbar}{2mc}\vec{\sigma}\vec{B} \right] \varphi$$

The important feature is appearance of MDM related to the spin $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$:

$$\vec{\mu}_{spin} = 2\,\frac{e}{2mc}\,\vec{S},$$

Now we add orbital MDM and spin MDM and obtain

$$\vec{\mu} = \vec{\mu}_{orb} + \vec{\mu}_{spin} = \frac{e}{2mc}(\vec{L} + 2\vec{S})$$

It is seen that the spin MDM enters with factor 2 compared to the orbital MDM.

Spin dipole moment

The spin MDM is intrinsic characteristic of a particle, like other particle properties, such as: mass, electric charge, lifetime, etc. We can write the spin MDM in the form

$$\vec{\mu} \equiv \vec{\mu}_{spin} = g \frac{e}{2mc} \vec{S} = \frac{g}{2} \frac{e\hbar}{2mc} \vec{\sigma}, \qquad with\ g = 2,$$

and g **is called** g**-factor, or gyromagnetic factor of a particle**.

Introduce the Bohr magneton

$$\mu_B \equiv \frac{|e|\hbar}{2mc},$$

which is for electron mass is equal to

$$\mu_B = \frac{|e|\hbar}{2m_e c} \approx 0.927 \cdot 10^{-20} \frac{erg}{Gauss}$$

.

What absolute value of MDM is measured? We take matrix element of the operator $\vec{\mu}$ between the Pauli spinors for spin projection $+1/2$ (on OZ axis) and obtain:

$$\frac{\mu}{\mu_B} = \frac{e}{|e|} \frac{g}{2}$$

For electron $e < 0$. We see that for a point-like electron g-factor is equal to 2. Is this true always?

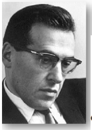
Electron g-factor: history

1948: Precise Measurement and Calculation

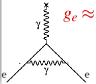
Kusch and Foley measure g_e

g_e = 2.00238 +/- 0.00006

Experiment (1948): Polykarp Kusch, H.M. Foley



1947 : QED

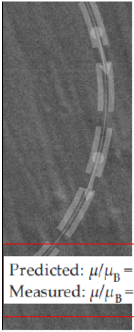


$$g_e \approx 2(1 + \frac{\alpha}{2\pi}) \approx 2.00232$$

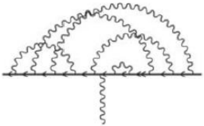
... and Feynman and Tomonaga ....

Theory: Julian Schwinger (1948, 1949)

Electron g-factor: experiment vs. theory for $\mu/\mu_B = g/2$

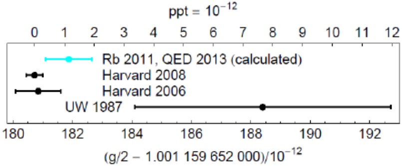


December 2013 Physics Today



from measured
fine structure constant

Predicted: $\mu/\mu_B = -1.001\,159\,652\,181\,78\,(77)$
Measured: $\mu/\mu_B = -1.001\,159\,652\,180\,73\,(28)$



Is theory in agreement with experiment?
Take difference of central values and compare with total error (standard deviation σ)

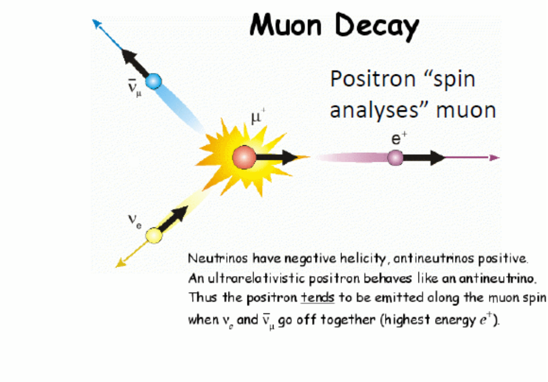
$$\Delta = \mu/\mu_B(exp) - \mu/\mu_B(theory) = (178 - 73)\times 10^{-14} = 105 \times 10^{-14},$$
$$\sigma = \sqrt{\sigma^2_{theory} + \sigma^2_{exp}} = \sqrt{77^2 + 28^2}\times 10^{-14} = 82 \times 10^{-14}$$

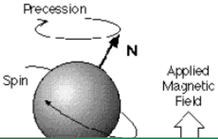
We see that $\Delta = 1.28\,\sigma$ which means that agreement for electron is indeed very good!

Muon g-factor: history

Muon is unstable ($\tau \approx 2 \times 10^{-6}$ s) and decays via

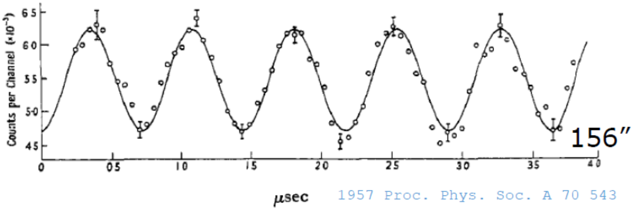
$$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$$





Muon g-factor: history

Garwin, Lederman, Weinrich 2.00+/-0.10 Phys Rev 105, 1415 (Jan 57) @ Columbia



1957 Proc. Phys. Soc. A 70 543

Figure 2. Time distribution of forward electrons from positive muons stopped in copper (87%) and carbon (13%). The magnetic field was 101·9 gauss. The exponential decay factor has been removed, and the first few points have been corrected for a slight non-linearity in the time analyser. Note the displaced zero

Experiments with a Polarized Muon Beam

By J. M. CASSELS, T. W. O'KEEFFE, M. RIGBY, A. M. WETHERELL AND J. R. WORMALD

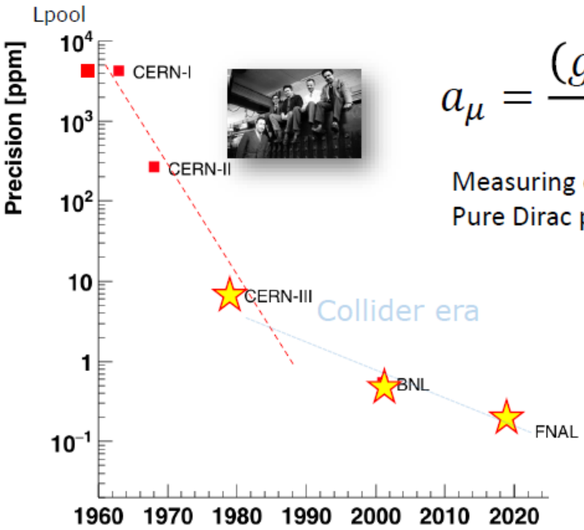
Nuclear Physics Research Laboratory, University of Liverpool

$g = 2.004 \pm 0.014$ (0.6%)

In 1959 CERN launched the g-2 experiment aimed at measuring the anomalous magnetic moment of the muon. The measures were studied using a magnet 83cm x 52cm x 10cm borrowed from the University of Liverpool.

In 1962 this precision had been whittled down to just 0.4%.

Muon $g-2$ measurements

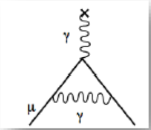


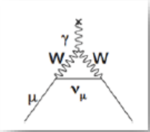
$$a_\mu = \frac{(g-2)}{2}$$

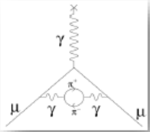
Measuring deviations from Pure Dirac prediction

Muon MDM: theory

$$a^{SM} = a^{QED} + a^{Weak} + a^{Hadronic}$$







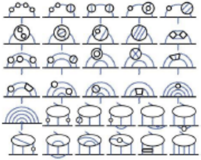
Vacuum Polarization

LO + NLO ...

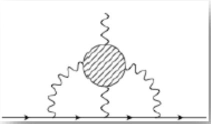
T. Aoyama, M. Hayakawa,
T. Kinoshita, M. Nio (PRLs, 2012)

Theory: 12,672 Feynman Diagrams

2.00231930436356 ± 0.00000000000154



~ 60% total SM uncertainty



Light by Light

~ 40% total SM uncertainty

Muon g-factor: experiment vs. theory for $(g/2-1)\times 10^{10}$

	2011		2017	*to be discussed
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[Phys. Rev. Lett. 109 (2012) 111808]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]*
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]*
	HLMNT11		**KNT17**	
LO HVP	694.91 (4.27)	$\longrightarrow$	692.23 **(2.54)**	this work*
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.83 (0.04)	this work*
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144] *
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659181.00 (3.62)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	28.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.9σ	this work

To compare theory and experiment calculate

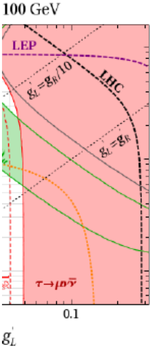
$$\Delta = \mu/\mu_B(exp) - \mu/\mu_B(theory) \approx 28.1 \times 10^{-9},$$

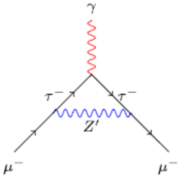
$$\sigma = \sqrt{\sigma^2_{theory} + \sigma^2_{exp}} = \sqrt{3.62^2 + 6.33^2} \times 10^{-9} = 7.3 \times 10^{-9}$$

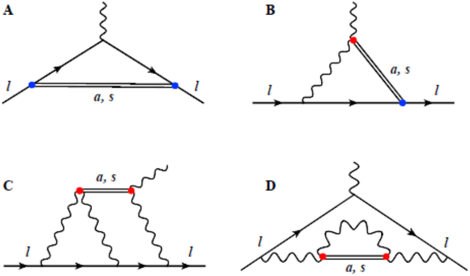
We see that $\Delta = 3.9\,\sigma$ which means a big disagreement!

Muon MDM: models of physics beyond Standard model

- light Z' can evade many searches involving electrons by non-standard couplings preferring heavy leptons (but see BaBar's direct search limits in a wide mass range, PRD 94 (2016) 011102), or invoke flavour off-diagonal Z' to evade constraints [Altmannshofer et al., PLB 762 (2016) 389]







- axion-like particle (ALP), contributing like π^0 in HLbL [Marciano et al., PRD 94 (2016) 115033]

- `dark photon' - like fifth force particle [Feng et al., PRL 117 (2016) 071803]

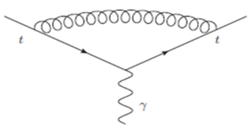
Because of the quark confinement one cannot measure MDM of the free quark, only for quarks inside baryons or mesons.

In general, for a quark

$$\mu_q = \frac{|e| Q_q \hbar}{2 m_q c} \frac{g_q}{2}$$

g_q - gyromagnetic factor, $Q_q = +2/3$ for quarks u, c, t and $Q_q = -1/3$ for quarks d, s, b.

For a point-like Dirac quark $g_q = 2$, however there are radiative corrections like for electron or muon, but with coupling constant α_s. For example, for the charm quark $\alpha_s(m_c) = 0.3378 \gg \alpha_{em} \approx 0.0073$.



$+$ higher orders in α_s.

For the charm quark [Grozin et al., 2008] radiative corrections up to 3 loops are

$$\frac{g_c}{2} - 1 = 0.03585 + 0.04 + 0.05685 + \mathcal{O}(\alpha_s^4(m_c)) = 0.1327$$

However, this result is not reliable since there is no convergence in α_s expansion.

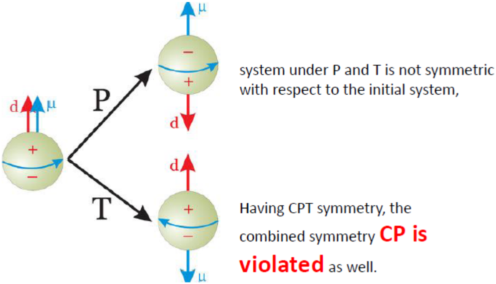
Electric dipole moment (EDM) of elementary particles

EDM is even more interesting and intriguing characteristic of particles.

$$\vec{\mu} \sim \vec{S}, \qquad \vec{d} \sim \vec{S}$$

But the polar vector $\vec{d}$ and axial vector of spin $\vec{S}$ have opposite properties with respect to space-reflection $\hat{P}$ and time-inversion $\hat{T}$ operations. This means that EDM is not zero only if $\hat{P}$ is violated and $\hat{T}$ is violated [L. Landau, 1957].

$\hat{P}$ is violated in weak interactions – this is not a surprise. But if $\hat{T}$ is violated, then due to the famous *CPT* theorem [W. Pauli, G. Luders, 1954], *CP* **symmetry should be also violated!**



Electric dipole moment (EDM) of particles

$\vec{\mu}$ is axial vector like magnetic field $\vec{B}$, and Hamiltonian is symmetric under $\hat{P}$ and $\hat{T}$. Interaction of $\vec{d}$ with electric field $\vec{E}$ violates both $\hat{P}$ and $\hat{T}$:

Magnetic dipole moment

$$\vec{\mu} = g\frac{Qe}{2m_\mu}\vec{s}$$

η is a dimensionless constant, analogous to g

Electric dipole moment

$$\vec{d}_\mu = \eta\frac{Qe}{2m_\mu c}\vec{s}$$

Hamiltonian for a fermion in B and E field

$$\hat{H} = -\vec{\mu}\cdot\vec{B} - \vec{d}\cdot\vec{E}$$

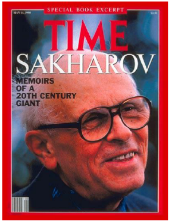
Transformation Properties

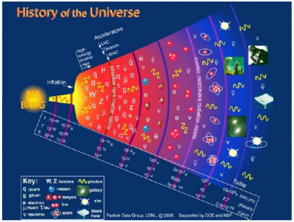
	$\vec{B}$	$\vec{E}$	$\vec{\mu}$	$\vec{d}$
C	-	-	-	-
P	+	-	+	+
T	-	+	-	-
CP	-	+	-	-
CPT	+	+	+	+

If CPT valid → EDM would violate CP

Electric dipole moment and CP violation

Why is CP symmetry violation so important? Because it is related to the problem of matter-antimatter asymmetry in the Universe.





"CP Symmetry Violation, C-Asymmetry, and Baryon Asymmetry of the Universe" *e"*. *Journal of Experimental and Theoretical Physics*. ***5****: 24–27. 1967*

Criteria of Sakharov:

1. Violation of CP symmetry. In fact, it is violated in the Standard model via CKM matrix, but the effect is too small – many orders of magnitude below what is needed.
2. Nonconservation of baryon number and leptonic numbers.
3. Time period in evolution when the Universe was out of equilibrium.

EDM of leptons and quarks

For leptons and quarks one can define EDM

$$d = \frac{|e|Q}{2m}\frac{\eta}{2}$$

where η is analogue of g-factor for MDM.
What is known at present? There are no direct measurements of EDM. Theoretically, d for leptons is not zero but extremely small (4-loop diagrams in Standard Model). It scales with mass, i.e.

$$d_\tau \sim d_e \frac{m_\tau}{m_e}$$

.

	d_{exp}, $\lvert e\rvert \cdot cm$	d_{theory}, $\lvert e\rvert \cdot cm$
electron	$< 0.9 \times 10^{-28}$	$\sim 10^{-38}$
muon	$(-0.1 \pm 0.9) \times 10^{-19}$	$\sim 2 \times 10^{-36}$
τ lepton	$(-0.22 - 0.45) \times 10^{-16}$	$\sim 3.5 \times 10^{-35}$
neutron	$< 3 \times 10^{-26}$	$(1-6) \times 10^{-32}$
charm quark	not known	$< 4.4 \times 10^{-17}$

Note that factor η for the charm quark is not too small, namely $\eta_c < 1.8 \times 10^{-2}$.

Baryons (spin $= 1/2, 3/2, \ldots$):

$$\begin{array}{llll} p = uud, & n = udd, & \Sigma^+ = uus, & \Sigma^0 = uds \ \ldots \\ \Sigma_c^+ = udc, & \Sigma_c^{++} = uuc, & \Xi_c^+ = usc, & \Xi_c^0 = dsc \ \ldots \\ \Xi_{cc}^+ = dcc, & \Xi_{cc}^{++} = ucc, & \Omega_{cc}^+ = scc & \ldots \end{array}$$

Total wave function in the quark constituent model is

$$\Psi = \psi_{flavor} \times \psi_{spin} \times \psi_{space} \times \psi_{color}$$

All baryons have the color wave function (singlet under $SU(3)_{color}$)

$$\psi_{color} = \frac{1}{\sqrt{6}}\, \epsilon^{\alpha\beta\gamma}\, q_\alpha q_\beta q_\gamma = \frac{1}{\sqrt{6}}\,(qqq - qqq + \ \ldots)$$

For the proton and neutron, e.g., the spin-flavor wave functions $\psi_{flavor} \times \psi_{spin}$ are

$$|p; \tfrac{1}{2}, +\tfrac{1}{2}\rangle = \frac{1}{\sqrt{6}}(2u_\uparrow u_\uparrow d_\downarrow - u_\uparrow u_\downarrow d_\uparrow - u_\downarrow u_\uparrow d_\uparrow),$$

$$|n; \tfrac{1}{2}, +\tfrac{1}{2}\rangle = \frac{1}{\sqrt{6}}(2d_\uparrow d_\uparrow u_\downarrow - d_\uparrow d_\downarrow u_\uparrow - d_\downarrow d_\uparrow u_\uparrow),$$

MDM of long-lived baryons with $\tau \sim 10^{-10}$ s

If we introduce nuclear magneton $\mu_N = \frac{e\hbar}{2m_p c}$ then for the long-lived baryons it is convenient to write MDM in units of μ_N, i.e. $\frac{\mu}{\mu_N} = \frac{g}{2}$. Then one obtains

Magnetic Moments of Baryons

Baryon	μ/μ_N (Experiment)		Quark model:	μ/μ_N
p	$+2.792\,847\,386$	$\pm\,0.000\,000\,063$	$(4\mu_u - \mu_d)/3$	—
n	$-1.913\,042\,75$	$\pm\,0.000\,000\,45$	$(4\mu_d - \mu_u)/3$	—
Λ^0	-0.613	$\pm\,0.004$	μ_s	—
Σ^+	$+2.458$	$\pm\,0.010$	$(4\mu_u - \mu_s)/3$	$+2.67$
Σ^0			$(2\mu_u + 2\mu_d - \mu_s)/3$	$+0.79$
$\Sigma^0 \to \Lambda^0$	-1.61	$\pm\,0.08$	$(\mu_d - \mu_u)/\sqrt{3}$	-1.63
Σ^-	-1.160	$\pm\,0.025$	$(4\mu_d - \mu_s)/3$	-1.09
Ξ^0	-1.250	$\pm\,0.014$	$(4\mu_s - \mu_u)/3$	-1.43
Ξ^-	$-0.650\,7$	$\pm\,0.002\,5$	$(4\mu_s - \mu_d)/3$	-0.49
Ω^-	-2.02	$\pm\,0.05$	$3\mu_s$	-1.84

Charmed baryons

Charmed baryons include at least one charm quark c with electric charge $2/3\,|e|$ and mass $m_c = 1.27 - 1.7$ GeV.

Baryon	Flavor content	$SU(3)_f$	Charm	Mass (MeV)	Cross section, μb fixed tar.	collider	Life-length $c\tau$, or width Γ
Λ_c^+	$[ud]c$	$\bar{3}$	1	2286.5 ± 0.1	10.13	758.1	$60.0 \pm 1.2\,\mu$m
Ξ_c^+	$[us]c$	$\bar{3}$	1	2467.9 ± 0.2	0.588	65.5	$132.5 \pm 7.8\,\mu$m
Ξ_c^0	$[ds]c$	$\bar{3}$	1	2470.9 ± 0.3	0.510	65.6	$33.6 \pm 3.6\,\mu$m
Σ_c^{++}	uuc	6	1	2454.0 ± 0.1	0.863	42.0	1.9 ± 0.1 MeV
Σ_c^+	$\{ud\}c$	6	1	2452.9 ± 0.4	0.697	42.2	< 4.6 MeV
Σ_c^0	ddc	6	1	2453.8 ± 0.1	0.461	41.6	1.8 ± 0.1 MeV
$\Xi_c^{\prime +}$	$\{us\}c$	6	1	2578.4 ± 0.5	0.083	6.3	–
$\Xi_c^{\prime 0}$	$\{ds\}c$	6	1	2579.2 ± 0.5	0.072	6.6	–
Ω_c^0	ssc	6	1	2695.2 ± 1.7	0.028	3.0	$80.3 \pm 10\,\mu$m
Ξ_{cc}^{++}	ccu	3	2	3621.4 ± 0.8	$< 10^{-4}$	$\sim 10^{-3}$	$76.7 \pm 10\,\mu$m
Ξ_{cc}^+	ccd	3	2	3518.9 ± 0.9	$< 10^{-4}$	$< 10^{-3}$	–
Ω_{cc}^+	ccs	3	2	–	$< 10^{-4}$	$\sim 10^{-3}$	–

We would like to study baryons which decay due to weak interaction with lifetime $\sim 10^{-13}$ s, or $c\,\tau \sim 100\ \mu$m.

Charmed baryons: wave function and magnetic moment

As examples of the wave function of charmed baryons,

$$|\Lambda_c^+; \tfrac{1}{2}, +\tfrac{1}{2}\rangle = \frac{1}{2}(u_\uparrow d_\downarrow c_\uparrow - u_\downarrow d_\uparrow c_\uparrow - d_\uparrow u_\downarrow c_\uparrow + d_\downarrow u_\uparrow c_\uparrow),$$

or

$$|\Sigma_c^+; \tfrac{1}{2}, +\tfrac{1}{2}\rangle = \frac{1}{2\sqrt{3}}(2u_\uparrow d_\uparrow c_\downarrow + 2d_\uparrow u_\uparrow c_\downarrow - u_\uparrow d_\downarrow c_\uparrow - d_\uparrow u_\downarrow c_\uparrow - u_\downarrow d_\uparrow c_\uparrow - d_\downarrow u_\uparrow c_\uparrow)$$

How to find MDM (or EDM) of the charm baryons?
We take matrix element between wave function of the MDM operator:

$$\mu = \langle B; \tfrac{1}{2}, +\tfrac{1}{2}|(\mu_1\sigma_{1z} + \mu_2\sigma_{2z} + \mu_3\sigma_{3z})|B; \tfrac{1}{2}, +\tfrac{1}{2}\rangle$$

where $\mu_i = \frac{|e|Q_i\hbar}{2m_i c}\,\frac{g_i}{2}$ is the magnetic moment of a quark ($i = 1, 2, 3$).

Magnetic moments of charmed baryons

In this way one obtains MDM of all charmed baryons

$$\mu_{\Lambda_c^+} = \mu_{\Xi_c^+} = \mu_{\Xi_c^0} = \mu_c,$$
$$\mu_{\Sigma_c^{++}} = \frac{1}{3}(4\mu_u - \mu_c), \quad \mu_{\Sigma_c^+} = \frac{1}{3}(2\mu_u + 2\mu_d - \mu_c), \quad \mu_{\Sigma_c^0} = \frac{1}{3}(4\mu_d - \mu_c),$$
$$\mu_{\Xi_c'^+} = \frac{1}{3}(2\mu_u + 2\mu_s - \mu_c), \qquad etc.\ for\ all$$

We observe, that MDM (and similarly for EDM) of some baryons are equal to the MDM (and EDM) of the charm quark:

$$\mu_{\Lambda_c^+} = \mu_c$$
$$d_{\Lambda_c^+} = d_c$$

which allows one to obtain information of the MDM and EDM of the charm quark.

Precession of spin in external magnetic and electric fields

How to measure MDM/EDM of particles which live so short time $\tau \sim 2 \times 10^{-13}$ s?

We need to accelerate it to increase its lifetime and distance it passes, $L = \gamma\, v\, \tau$, where $\gamma = (1 - \vec{v}^2/c^2)^{-1/2} = E/m$ is Lorentz factor.
For the LHC, the energy $E \sim$ *a few TeV*, then $v \approx c$, $\gamma \sim 10^3$ and the length is macroscopic, $L \sim 10$ cm.

Then one can use phenomenon of spin precession in external fields.
In the rest frame of particle the vector of spin satisfies eq-n

$$\frac{d\vec{S}}{d\tau} = \vec{\mu} \times \vec{B}^\star + \vec{d} \times \vec{E}^\star,$$

where $\vec{B}^\star$ and $\vec{E}^\star$ are the magnetic and electric fields in the rest frame and τ is the proper time ("собственное время"). Of course, $\vec{\mu} \sim \vec{S}$ and $\vec{d} \sim \vec{S}$.
For example, if $\vec{E}^\star = 0$ and $\vec{B}^\star \neq 0$, then the spin rotates around the magnetic field with the angular velocity

$$\vec{\omega} = -\frac{e\vec{B}^\star}{mc}\,\frac{g}{2}, \qquad \omega = \frac{eB^\star}{mc}\,\frac{g}{2}$$

Precession of spin in external fields

We need description of spin precession in external static fields for a ultrarelativistic fermion.

- L.H. Thomas, Nature 117, 514 (1926)
- V. Bargmann, L. Michel, V.L. Telegdi, Phys. Rev. Lett. 2, 435 (1959)
- V.B. Beresteckii, E.M. Lifshitz, L.P. Pitaevskii, "Quantum electrodynamics", sec. 41, Pergamon Press, 1982
- J.D. Jackson, "Classical electrodynamics", sec. 11.11, John Wiley, 3rd ed., 1999
- V. Lyuboshits, Yad. Fiz. 31 (1980) 986; I. Kim, Nucl. Phys. B229 (1983) 251.
- V.G. Baryshevsky, Phys. Lett. B757 (2016) 426.

One can write equation in Laboratory frame, where the particle moves, by transforming the fields and time t to the Lab frame:

$$\vec{B}^\star = \gamma(\vec{B} - \frac{\vec{v}\times\vec{E}}{c}) - \frac{\gamma^2}{1+\gamma}\frac{\vec{v}(\vec{v}\vec{B})}{c^2},$$

$$\vec{E}^\star = \gamma(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}) - \frac{\gamma^2}{1+\gamma}\frac{\vec{v}(\vec{v}\vec{E})}{c^2}$$

and $dt = \gamma d\tau$. Then the precession equation in the Lab reads

$$\frac{d\vec{S}}{dt} = \frac{d\vec{S}}{dt}|_{rest} + \frac{\gamma^2}{1+\gamma}\frac{\vec{S}\times(\vec{v}\times\vec{a})}{c^2}$$

where the 2nd term is the so-called Thomas correction related to noninertial frame of particle.

Precession of spin in external fields

Acceleration of particle is

$$\frac{d\vec{v}}{dt} = \frac{q}{m\gamma}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c} - \frac{\vec{v}(\vec{v}E)}{c^2}\right)$$

$$= \vec{\omega}_0 \times \vec{v} + \frac{q}{m\gamma}\frac{1}{\gamma^2-1}\frac{\vec{v}(\vec{v}E)}{c^2},$$

$$\vec{\omega}_0 = \frac{q}{mc\gamma}\left(\frac{\gamma^2}{\gamma^2-1}\frac{\vec{v}\times\vec{E}}{c} - \vec{B}\right)$$

and $\vec{\omega}_0$ is angular velocity of rotation of velocity (or trajectory) of a particle.
The final equations for the spin precession are

$$\frac{d\vec{S}}{dt} = \vec{\Omega}\times\vec{S}, \qquad \vec{\Omega} = \vec{\omega}_{MDM} + \vec{\omega}_{EDM},$$

$$\vec{\omega}_{MDM} = \vec{\omega}_B + \vec{\omega}_E,$$

$$\vec{\omega}_B = -\frac{q}{mc}\left[\left(\frac{g}{2} - 1 + \frac{1}{\gamma}\right)\vec{B} - \left(\frac{g}{2} - 1\right)\frac{\gamma}{1+\gamma}\frac{\vec{v}\,(\vec{B}\,\vec{v})}{c^2}\right],$$

$$\vec{\omega}_E = -\frac{q}{mc}\left(\frac{g}{2} - \frac{\gamma}{1+\gamma}\right)\frac{\vec{E}\times\vec{v}}{c},$$

$$\vec{\omega}_{EDM} = -\frac{\eta q}{2mc}\left[\vec{E} + \frac{\vec{v}\times\vec{B}}{c} - \frac{\gamma}{1+\gamma}\frac{\vec{v}(\vec{v}E)}{c^2}\right]$$

Spin precession in electric field of a bent crystal

In crystal there is only electric field, such that $\vec{E} \perp \vec{v}$. Then the energy of particle is conserved, so it moves with constant velocity.
Choose the components of electric field and velocity:

$$\vec{E} = (E,\, 0,\, 0), \qquad \vec{v} = (0,\, 0,\, v),$$

then momentum of particle rotates with angular velocity

$$\vec{\omega}_0 = (0,\, -\omega_0,\, 0), \qquad \omega_0 = \frac{qE}{m\gamma v} = \frac{v}{R},$$

where R is the curvature of a crystal.
Then we find the spin rotation velocity due to MDM

$$\vec{\omega}_{MDM} = (0,\, \omega_{MDM},\, 0), \qquad \omega_{MDM} = \gamma\omega_0(\frac{g}{2} - 1 - \frac{g}{2\gamma^2} + \frac{1}{\gamma})$$

and due to EDM

$$\vec{\omega}_{EDM} = (\omega_{EDM},\, 0,\, 0), \qquad \omega_{EDM} = \gamma\omega_0\, \frac{\eta q}{2c}$$

So that $\vec{\Omega} = (\omega_{EDM},\, -\omega_{MDM},\, 0)$ is in some direction in OXY plane.

Precession of spin in electric field of the bent crystal

Integration leads to relations for the angle of rotation of polarization

$$\vec{\Phi} = (\theta',\, -\theta,\, 0),$$
$$\theta = \gamma\, \theta_0\, (\kappa - \frac{g}{2\gamma^2} + \frac{1}{\gamma}), \qquad \theta' = \gamma\, \theta_0\, \frac{\eta\, v}{2c}, \tag{1}$$

where $\theta_0 = \frac{L}{R}$, L is the arc length that baryon passes in the channeling regime and $\kappa = \frac{g}{2} - 1$ is anomalous magnetic moment.
The polarization vector rotates around the unit vector $\vec{n}$ by the angle Φ

$$\vec{n} = (\frac{\theta'}{\Phi},\, -\frac{\theta}{\Phi},\, 0), \qquad \Phi = \sqrt{\theta^2 + \theta'^2}$$

What is typical rotation angle θ_0 of the particle with velocity $v \approx c$?

$$\theta_0 = \frac{L}{R} = \frac{v\, t}{R} = \frac{v\, \gamma\, \tau}{R} \sim \frac{10\, cm}{10\, m} \sim 10\, \mathrm{m}rad \sim 0.6^o$$

Rotation of spin in electric field of bent crystal

After particle passing the crystal the polarization vector acquires the components which depend on initial polarization and rotation angles θ and θ'.
If crystal is oriented perpendicular to initial polarization and $\theta' \ll \theta$, then

$$\vec{\mathcal{P}}_{in} = \mathcal{P}\,(1,\,0,\,0) \quad \Longrightarrow \vec{\mathcal{P}}_{fin} \approx \mathcal{P}\,(\cos\theta,\,0,\,\sin\theta)$$

which can be used to find $\theta \sim \kappa$ (MDM of the particle).
If crystal is rotated then component OY of polarization appears

$$\vec{\mathcal{P}}_{in} = \mathcal{P}'\,(0,\,1,\,0) \quad \Longrightarrow \vec{\mathcal{P}}_{fin} \approx \mathcal{P}'\,\left(\frac{\theta'}{\theta}(\cos\theta - 1),\,1,\,\frac{\theta'}{\theta}\sin\theta\right)$$

which can be convenient for measurement of $\theta' \sim \eta$ (EDM).
For example, if particle rotates by the angle $\theta_0 \sim 0.6^o$, $\gamma \sim 10^3$, and $\kappa \sim 0.01$, then

$$\theta \approx \gamma\,(\frac{g}{2} - 1)\,\theta_0 \sim 10^3 \times 0.6^o \times 0.01 \approx 6^o$$

Even small deviation of g-factor from 2 will be enhanced by the big Lorentz factor. If we measure θ and know θ_0, then g-factor can be found.

Our proposal

Any direct measurement of MDM/EDM of short-lived charmed baryons, beauty baryons, τ lepton would be the first one.

Collaboration: Orsay (LAL and Paris-Sud University) & Kharkiv (KIPT and V.N. Karazin University) & Kyiv (Taras Shevchenko University) & CERN & Rome (INFN)

- A.S. Fomin, A.Yu. Korchin, A. Stocchi, O.A. Bezshyyko, L. Burmistrov, S.P. Fomin, I.V. Kirillin, L. Massacrier, A. Natochii, P. Robbe, W. Scandale, N.F. Shul'ga, *JHEP 08 (2017) 120*
- A.S. Fomin, A.Yu. Korchin, A. Stocchi, S. Barsuk, P. Robbe, *JHEP 03 (2019) 156*
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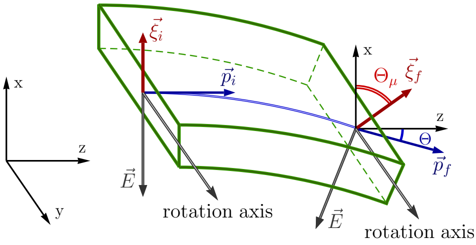
- D. Mirarchi, A.S. Fomin, S. Redaelli, W. Scandale, arXiv:1906.08551 [physics.acc-ph].
- A.S. Fomin et al. *"Physics Beyond Colliders" Annual Workshop, CERN, Geneva, 21-22 Nov 2017*
- A.S. Fomin et al. *"Channeling 2018", ISCHIA (NAPLES-Italy), Sep 23-28, 2018*
- A.S. Fomin et al. *"Electromagnetic dipole moments of unstable particles", Milano, Italy, 2-4 October 2019*
- A.S. Fomin et al. *FTE@LHC and NLOAccess STRONG 2020 joint kick-off Meeting, CERN, Geneva, 7-8 November 2019*
- ...

Schematically rotation of polarization in a bent crystal



The gradient of the inter-plane electric field of a silicon crystal reaches the maximum value about 5 GeV/cm. This corresponds to the induced magnetic field in the instantaneous rest frame of a particle $\vec{B}^* = \gamma(\frac{\vec{v}}{c} \times \vec{E}) \sim 10^3$ Tesla, if the particle moves with relativistic energies $\sim$ TeV.

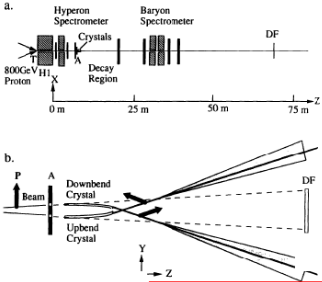
With Lorentz factor $\gamma \sim 10^3$ the particle can move about ~ 10 cm in the crystal before decaying to observed particles.

Fermilab experiment with Σ^+ (lifetime $\approx 10^{-10}$ s) in 1992

E761 Collaboration. Measurement of the Σ+ magnetic moment - 1

VOLUME 69, NUMBER 23 PHYSICAL REVIEW LETTERS 7 DECEMBER 1992

First Observation of Magnetic Moment Precession of Channeled Particles in Bent Crystals



Proton (800GeV/c) + Cu → Σ^+ n particles

$\Sigma^+ \to p\, \pi^0$

As illustrated in Fig. 1, a vertically polarized Σ^+ beam [14] was produced by directing the Fermilab Proton Center extracted 800-GeV/c proton beam onto a Cu target (T). The resulting Σ^+ were produced alternately at a +3.7- or −3.7-mrad horizontal targeting angle relative to the incident proton beam direction. This allowed the polarization direction to be periodically reversed. The mean

The two bending crystals. Each crystal precess the channelled particle's spin in opposite direction

The deflection of the channeled particles was measured to be $\omega = 1.649 \pm 0.043$ and -1.649 ± 0.030 mrad for the up- and down-bending crystals, respectively. For 375-GeV/c Σ^+ this corresponds to an effective magnetic field of $B_x \approx 45$ T in the crystals. The magnetic moment [6] of the Σ^+ should precess by $\varphi \approx 1$ rad in such a field.

Concept of experiment at CERN

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Feasibility of measuring the magnetic dipole moments of the charm baryons at the LHC using bent crystals

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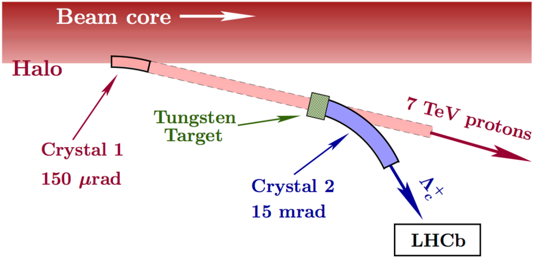


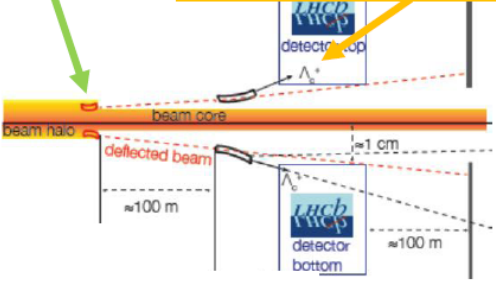
The proposed experiment at CERN (slide of A. Stocchi)

The proposed experiment in LHC

Studies mainly done looking at installing the double crystal system downstream to LHCb detector

Bent crystals inserted in the LHC beam halo deflected protons into an absorber
Collimation Team and UA9 collaboration, Phys. Lett. B **758**, 129 (2016)

HERE ONE OF THE VERY CRITICAL POINT
$L\approx 10$ cm, $\vartheta_C \approx 15$ mrad



Polarisation ($\mathcal{P}$) of Λc and weak asymmetry decay parameter (α)

$\mathcal{P}$ (Λc)

The polarisation $\mathcal{P}$ of Λc has not been yet measured precisely.

There are some old experiment and the indicative values are

$\mathcal{P}(\Lambda_c)$ ~[0.4-0.6] ($\mathcal{P}(\Lambda_c)$ = 0.6 (e.g. Bis-2)

$\mathcal{P}$ (Λc) ~ 0.6
To be also measured
by this experiment

α

The parameter α is decay dependent

channel	Br		α
($\Lambda_c \to \Lambda\pi$) X Br($\Lambda \to$p π)	1,07% x 64 % ~ 0,007	~ 1	0.59
($\Lambda_c \to \Lambda\pi$) X Br($\Lambda \to$n π^0)	1,07% x 35,8 % ~ 0,004	~ 0.6	0.59
($\Lambda_c \to \Sigma^+\pi^0$) X Br($\Sigma^+ \to$p π^0)	1,00% x 51,5 % ~ 0,005	~ 0.7	0.44
($\Lambda_c \to \Sigma^+\pi^0$) X Br($\Sigma^+ \to$n π^+)	1,00% x 48,3 % ~ 0,005	~ 0.6	~0
($\Lambda_c \to \Lambda e\nu$) X Br($\Lambda \to$p π)	2,00% x 64 % ~ 0,0128	~ 1.8	0.60
($\Lambda_c \to \Lambda\mu\nu$) X Br($\Lambda \to$p π)	2,00% x 64 % ~ 0,0128	~ 1.8	0.60
($\Lambda_c \to$ p $K^-\pi^+$)	**5,00% ~ 0,05**	**~12.5**	**not known**

α
input parameter

For the numerical study
we use
$\mathcal{P}$ (Λc)x α~ 0.6x0.59 ~ 0.35

Two observations :

1) Consider that the sensitivity of the analysis goes as $(\mathcal{P} \times \alpha)^2$
2) More decay channels can be use. In particular if the α parameter of $\Lambda_c \to$ p $K^-\pi^+$ decay mode is measured and happened to be large, it would allow to give access to much larger statistics. **Possible at LHCb !**

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Feasibility of measurement for τ-lepton

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Feasibility of τ-lepton electromagnetic dipole moments measurement using bent crystal at the LHC

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JHEP03(

Tau-lepton is a short-lived fermion with lifetime 2.9×10^{-13} s.
Its MDM and EDM have never been measured.
How to produce polarized τ? One can take charmed-strange meson $D_s^+ = (c\,\bar{s})$ with sizable branching rate (5.5%) of decay

$$D_s^+ \to W^+(virtual) \to \tau^+ + \nu_\tau$$

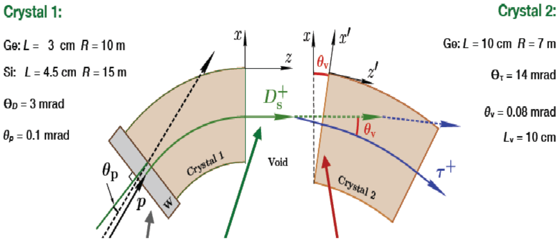
D_s mesons are produced at the LHC in *pp* collisions with very high energies, of a few TeV, and subsequently decay to **100 % polarized τ leptons.**

Measurement of MDM and EDM for τ-lepton

τ leptons can be directed into a bent crystal, get in the channeling regime, and the direction of τ polarization after the spin precession in the crystal can be determined from the angular analysis of its decay products. Schematically, the whole process is

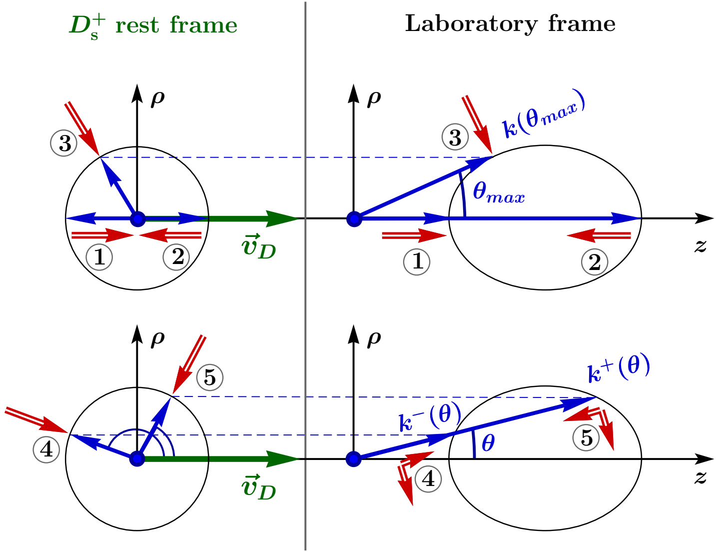
$$p\,p \to D_s^+ X \to \tau^+ Y \to \tau^+ \text{ in a crystal} \to \pi^+\pi^+\pi^-\bar{\nu}_\tau .$$

Double-crystal setup is proposed by Alexey Fomin. Optimal parameters: 1st crystal – silicon (L = 4.5 cm, R = 15 m) or germanium (L = 3 cm, R = 10 m); 2nd crystal – germanium (L = 10 cm, R = 7 m).



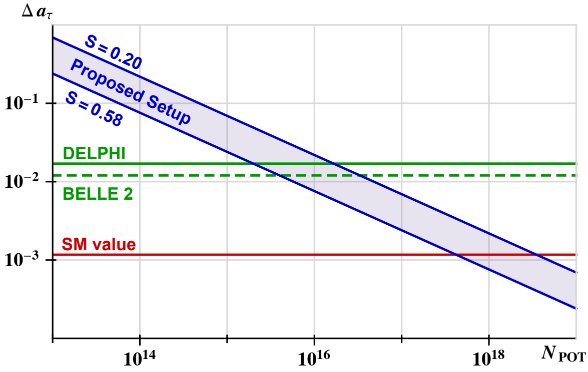
Polarization of τ in the weak decay $D_s^+ \to \tau^+ \nu_\tau$

Behavior of polarization vector of τ is complicated, depending on kinematics.



What can we expect in future measurements?

Absolute statistical error of the measured anomalous MDM of the τ lepton $a_\tau = \frac{1}{2}(g_\tau - 2)$ as a function of the total number of protons on target (N_{POT}).
The green lines show the limits obtained by the DELPHI collaboration ($\gamma\gamma \to \tau^+\tau^-$, LEP) and expected for the future experiment at BELLE 2, Japan ($\tau^- \to \ell^- \nu_\tau \bar{\nu}_\ell \gamma$). The red line – the Standard model prediction [Eidelman, Passera, 2007].



N_{POT} means "number of protons colliding with target".

Acknowledgement

Thank you for attention!