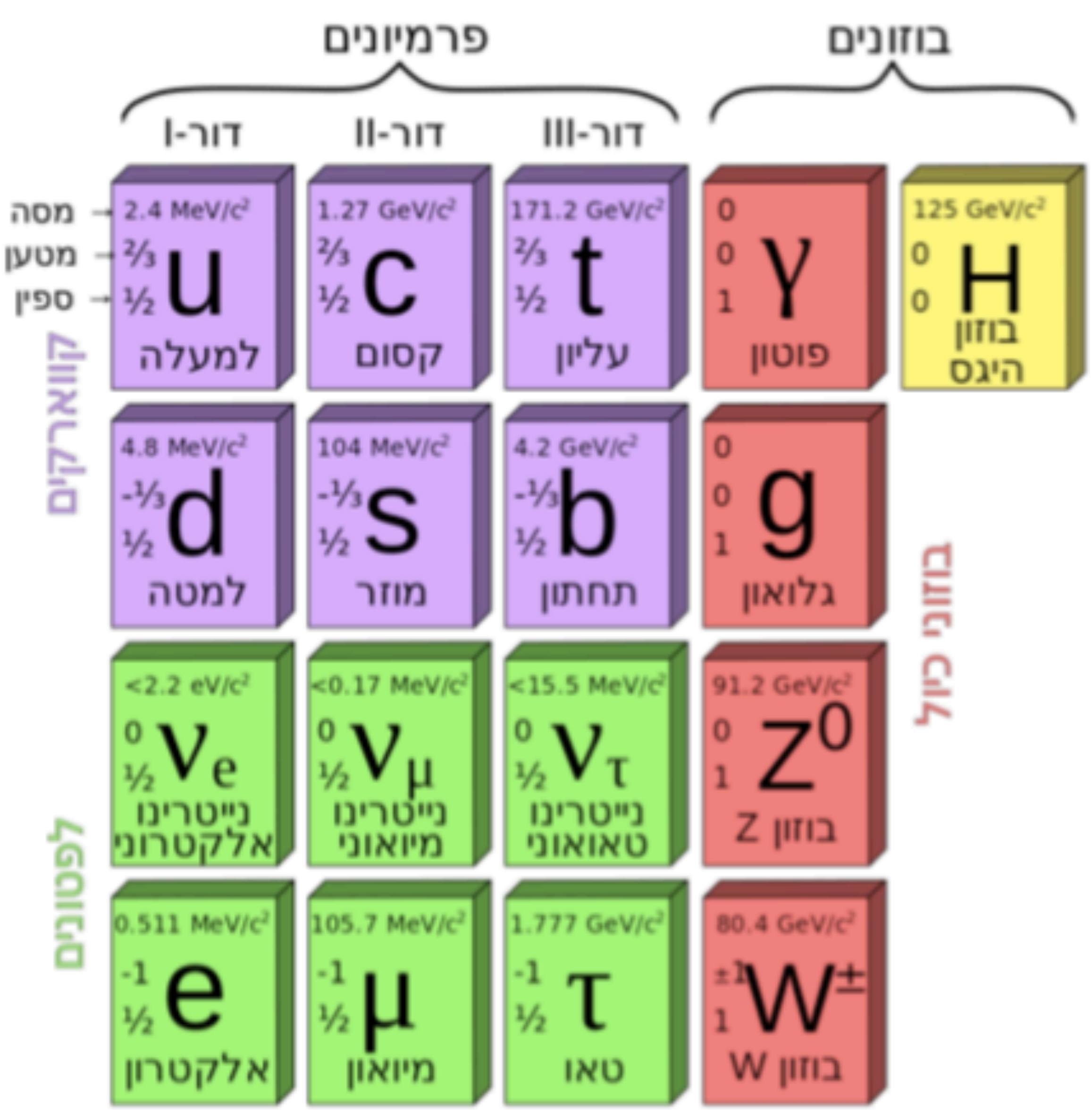
The Standard Model

The SM in a nutshell



- $SU(3)_c$ x $SU(2)_L$ x $U(1)_Y$ gauge symmetries.
- Matter is organised in chiral multiplets of the fundamental representation of the gauge groups.
- The SU(2) x U(1) symmetry is spontaneously broken to EM.
- Yukawa interactions are present that lead to fermion masses and CP violation.
- Neutrino masses can be accommodated in two distinct ways.
- Anomaly free.
- Renormalisable = valid to "arbitrary" high scales.

$SU(2)_L \times U(1)_Y$

Experimental evidence, such as charged weak currents couple only with left-handed fermions, the existence of a massless photon and a neutral $Z\ldots$, the electroweak group is chosen to be $SU(2)_L\times U(1)_Y$.

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \qquad \psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi \qquad \psi = \psi_L + \psi_R$$

$$L_L \equiv \frac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu_e \\ e \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad e_R \equiv \frac{1}{2}(1+\gamma_5)e$$

- $SU(2)_L$: weak isospin group. Three generators $\Longrightarrow$ three gauge bosons: W^1, W^2 and W^3, with gauge coupling g. The generators for doublets are $T^a = \sigma^a/2$, where σ^a are the 3 Pauli matrices (when acting on the gauge singlet e_R and ν_R, $T^a \equiv 0$).
- $U(1)_Y$: weak hypercharge Y. One gauge boson B with gauge coupling g'.
  One generator (charge) $Y(\psi)$, whose value depends on the corresponding field.

$SU(2)_L \times U(1)_Y$

Following the gauging recipe (for one generation of leptons. Quarks work the same way)

$$\mathcal{L}_\psi = i\,\bar{L}_L \not{D}\, L_L + i\,\bar{\nu}_{eR} \not{D}\, \nu_{eR} + i\,\bar{e}_R \not{D}\, e_R$$

where

$$D^\mu = \partial^\mu - igW_i^\mu T^i - ig'\frac{Y(\psi)}{2}B^\mu \qquad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0 \qquad i = 1, 2, 3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\begin{aligned}
\mathcal{L}_{kin} &= i\,\bar{L}_L \not{\partial}\, L_L + i\,\bar{\nu}_{eR} \not{\partial}\, \nu_{eR} + i\,\bar{e}_R \not{\partial}\, e_R \\
\mathcal{L}_{CC} &= g\,W_\mu^1\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_1}{2}\,L_L + g\,W_\mu^2\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_2}{2}\,L_L = \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{L}_L\,\gamma^\mu\,\sigma^+\,L_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{L}_L\,\gamma^\mu\,\sigma^-\,L_L \\
&= \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_L\,\gamma^\mu\,e_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \\
\mathcal{L}_{NC} &= \frac{g}{2}\,W_\mu^3\left[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L\right] + \frac{g'}{2}\,B_\mu\Big[Y(L)\left(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L\right) \\
&\quad + Y(\nu_{eR})\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y(e_R)\,\bar{e}_R\,\gamma^\mu\,e_R\Big]
\end{aligned}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right) \qquad \sigma^\pm = \frac{1}{2}\left(\sigma^1 \pm i\sigma^2\right)$$

$SU(2)_L \times U(1)_Y$

We perform a rotation of an angle θ_W, the Weinberg angle, in the space of the two neutral gauge fields (W_μ^3 and B_μ). We use an orthogonal transformation in order to keep the kinetic terms diagonal in the vector fields

$$\begin{aligned} B_\mu &= A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\ W_\mu^3 &= A_\mu \sin\theta_W + Z_\mu \cos\theta_W \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{NC} &= \bar{\Psi}\gamma^\mu \left[g \, \sin\theta_W \, \mathcal{T}_3 + g' \, \cos\theta_W \, \frac{\mathcal{Y}}{2} \right] \Psi \, A_\mu + \bar{\Psi}\gamma^\mu \left[g \, \cos\theta_W \, \mathcal{T}_3 - g' \, \sin\theta_W \, \frac{\mathcal{Y}}{2} \right] \Psi \, Z_\mu \\ &= e \, \bar{\Psi}\gamma^\mu \mathcal{Q} \Psi \, A_\mu + \bar{\Psi}\gamma^\mu \mathcal{Q}_Z \Psi \, Z_\mu \end{aligned}$$

where $\mathcal{Q}_Z$ is a diagonal matrix given by

$$\mathcal{Q}_Z = \frac{e}{\cos\theta_W \, \sin\theta_W} \left(\mathcal{T}_3 - \mathcal{Q} \sin^2\theta_W \right)$$

SM charge assignments

				$SU(3)$	$SU(2)$	$U(1)_Y$	$Q = T_3 + \frac{Y}{2}$
$Q^i_L =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{3}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u^i_R =$	u_R	c_R	t_R	3	1	$\frac{4}{3}$	$\frac{2}{3}$
$d^i_R =$	d_R	s_R	b_R	3	1	$-\frac{2}{3}$	$-\frac{1}{3}$
$L^i_L =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	-1	0 -1
$e^i_R =$	e_R	μ_R	τ_R	1	1	-2	-1
$\nu^i_R =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

Masses

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any mass terms for $W^\pm$ and Z.

Mass terms for gauge bosons

$$\mathcal{L}_{mass} = \frac{1}{2} m_A^2\, A_\mu\, A^\mu$$

are not invariant under a gauge transformation

$$A^\mu \;\rightarrow\; U(x) \left(A^\mu + \frac{i}{g} \partial^\mu \right) U^{-1}(x)$$

However, the gauge bosons of weak interactions are massive (short range of weak interactions).

BEH mechanism

We give mass to the gauge bosons through the Brout-Englert-Higgs mechanism: generate mass terms from the kinetic energy term of a scalar doublet field Φ that undergoes a broken-symmetry process.

Introduce a complex scalar doublet: four scalar real fields

$$
\begin{aligned}
\Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad\qquad Y(\Phi) = 1 \\
\mathcal{L}_{\text{Higgs}} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right) \\
D^\mu &= \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' \frac{Y(\Phi)}{2} B^\mu \\
V\left(\Phi^\dagger \Phi\right) &= -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2, \qquad\qquad \mu^2, \lambda > 0
\end{aligned}
$$

- The reason why $Y(\Phi) = 1$ is not to break electric-charge conservation.
- Charge assignment for the Higgs doublet through $Q = T_3 + Y/2$. The potential has a minimum in correspondence of

$$
|\Phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}
$$

BEH mechanism

Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i\theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an $\mathrm{SU}(2)_L$ gauge transformation

$$\Phi(x) \to \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i\theta^i(x)}{v}\right]$.

This gauge choice is called unitary gauge, and is equivalent to absorbing the Goldstone modes $\theta^i(x)$. Three would-be Goldstone bosons "eaten up" by three vector bosons $(W^\pm, Z)$ that acquire mass. This is why we introduced a complex scalar doublet (four elementary fields).

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

The Higgs potential

The scalar potential

$$V\left(\Phi^\dagger\Phi\right) = -\mu^2\Phi^\dagger\Phi \; + \; \lambda\left(\Phi^\dagger\Phi\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

becomes

$$V \;=\; \frac{1}{2}\left(2\lambda v^2\right)H^2 \; + \; \lambda v H^3 \; + \; \frac{\lambda}{4}H^4 + \text{const}$$

- the scalar field H gets a mass

$$m_H^2 = 2\lambda v^2 \qquad\qquad v^2 = \mu^2/\lambda$$

- there is a term of cubic and quartic self-coupling.

Note: this means that $\lambda_3 = \lambda_4 = \lambda$ in the SM. To have (independent) deviations of the trilinear or quadrilinear, one needs to deform the potential with a BSM hypothesis.

Vector boson masses and couplings

$$(D^\mu \Phi)^\dagger D_\mu \Phi = \frac{1}{2}\partial^\mu H \partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu +} W^-_\mu + \frac{1}{2}\frac{(g^2+g'^2)\,v^2}{4} Z^\mu Z_\mu\right]\left(1+\frac{H}{v}\right)^2$$

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad m_Z^2 = \frac{(g^2+g'^2)\,v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

  From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22 \text{ GeV}$$

- the photon stays massless
- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W^+_\mu W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv g m_W W^+_\mu W^{-\mu} H + \frac{1}{2}\frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Fermion masses and couplings

A direct mass term is not invariant under $SU(2)_L$ or $U(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij} \bar{Q}_L^{\prime i} \Phi d_R^{\prime j} - \Gamma_d^{ij*} \bar{d}_R^{\prime i} \Phi^\dagger Q_L^{\prime j} \\ &\quad -\Gamma_u^{ij} \bar{Q}_L^{\prime i} \Phi_c u_R^{\prime j} + \text{h.c.} \\ &\quad -\Gamma_e^{ij} \bar{L}_L^i \Phi e_R^j + \text{h.c.} \end{aligned}$$

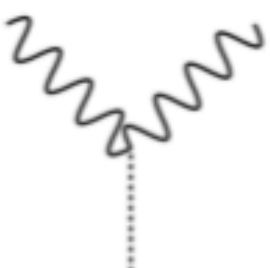
$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$$M^{ij} = \Gamma^{ij} \frac{v}{\sqrt{2}}$$

Higgs couplings

$$i\, m_f / v$$



$$i g m_W g_{\mu\nu} = 2i\, v g_{\mu\nu} \cdot m_W^2 / v^2$$

$$ig \frac{m_Z}{\cos\theta_W} g_{\mu\nu} = 2iv g_{\mu\nu} \cdot m_Z^2 / v^2$$

$$-3\, iv \cdot m_h^2 / v^2$$

1. The coupling to fermions is proportional to the mass.

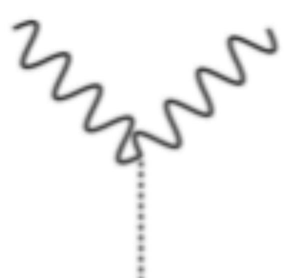
2. The coupling to bosons is proportional to the mass squared.

3. Four-point couplings HHVV and HHHH are also predicted from the gauge symmetry and the structure of the Higgs potential.

4. Couplings to photons and gluons are loop (Vs and quarks) induced.

Higgs couplings

$$i\, m_f/v$$



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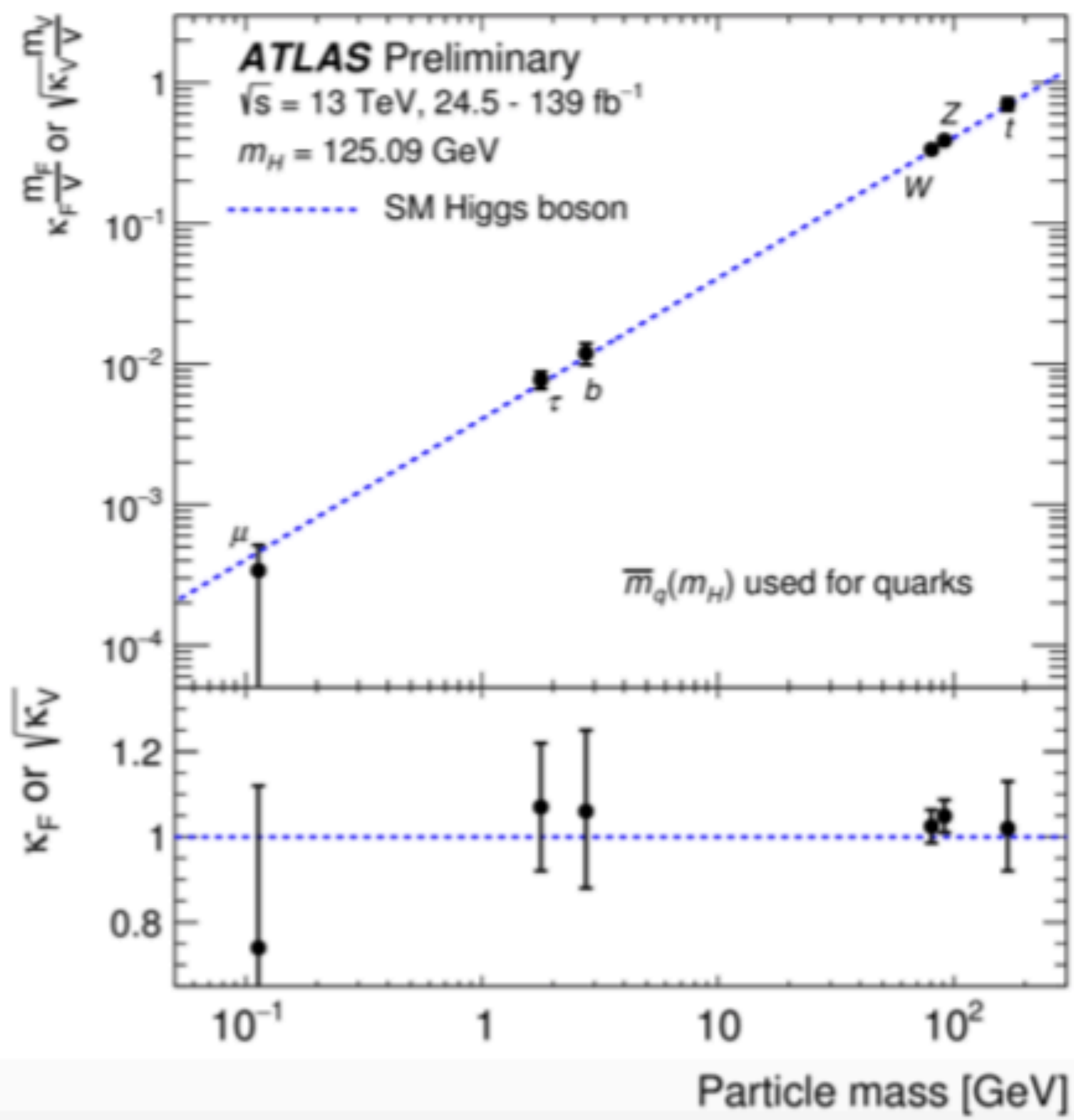


Naturalness

Apart from the considerations made up to now, the SM must be considered as an effective low-energy theory: at very high energy new phenomena take place that are not described by the SM (gravitation is an obvious example) $\Longrightarrow$ other scales have to be considered.

Why the weak scale ($\sim 10^2$ GeV) is much smaller than other relevant scales, such as the Planck mass ($\approx 10^{19}$ GeV) or the unification scale ($\approx 10^{16}$ GeV) (or why the Planck scale is so high with respect to the weak scale $\Longrightarrow$ extra dimensions)?

This is the **hierarchy problem**.

And this problem is especially difficult to solve in the SM because of the un-naturalness of the Higgs boson mass.

As we have seen and as the experimental data suggest, the Higgs boson mass is of the same order of the weak scale. However, it's not naturally small, in the sense that there is no approximate symmetry that prevent it from receiving large radiative corrections.

As a consequence, it naturally tends to become as heavy as the heaviest degree of freedom in the underlying theory (Planck mass, unification scale).

Naturalness in the SM

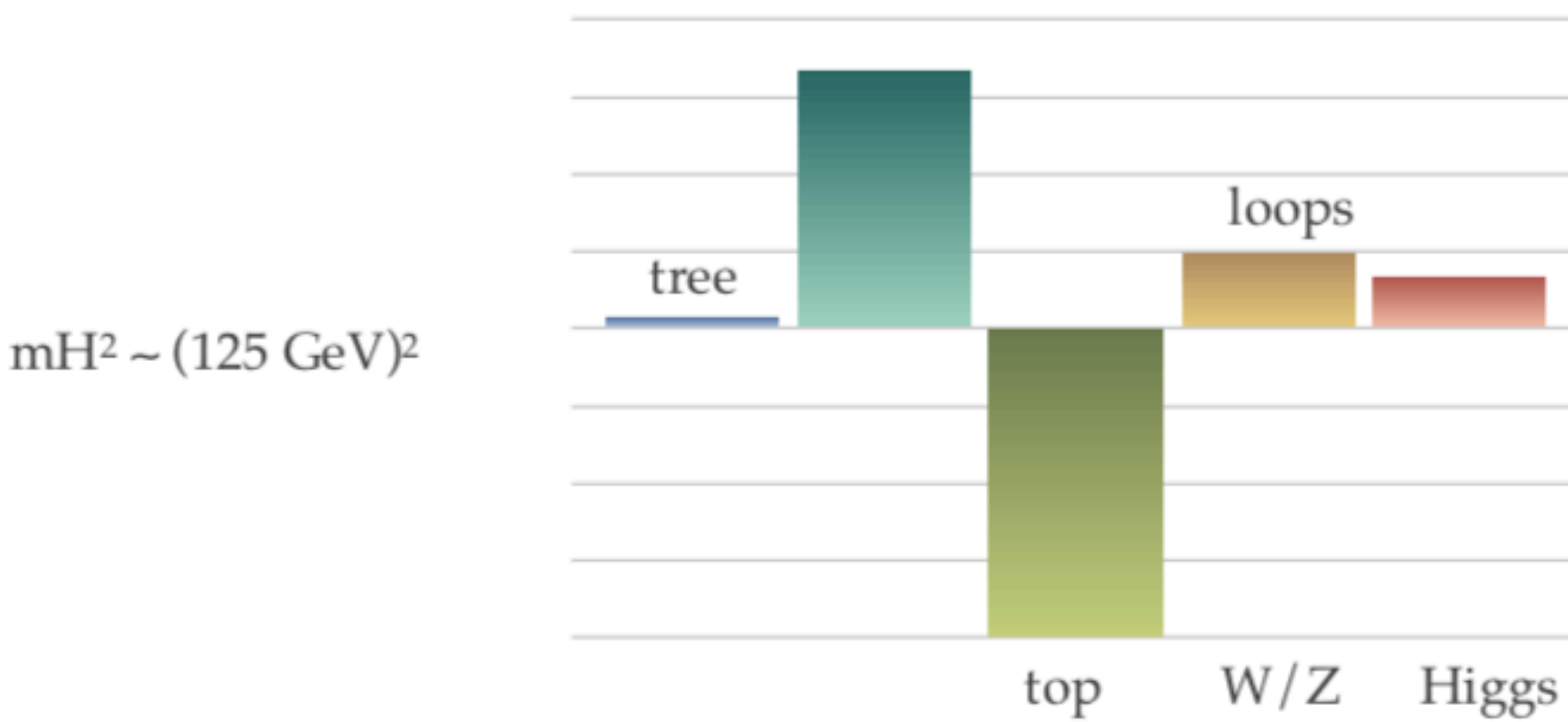
The Higgs mass is renormalised additively. Using a cutoff the regularization :

$$m_H^2 = m_{H0}^2 \quad - \frac{3}{8\pi^2} y_t \Lambda^2 \quad + \frac{1}{16\pi^2} g^2 \Lambda^2 \quad + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

Putting numbers, one gets:

$$(125\,\text{GeV})^2 = m_{H0}^2 + \left[-(2\,\text{TeV})^2 + (700\,\text{GeV})^2 + (500\,\text{GeV})^2\right] \left(\frac{\Lambda}{10\text{TeV}}\right)^2$$

Naturalness in the SM



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Definition of naturalness: less than 90% cancellation:

$\Lambda_t < 3\,\text{TeV}$ ⇒ top partners must be "light"

The Higgs boson

1. The scalar excitation of the Higgs field with respect of the EWSB vacuum.

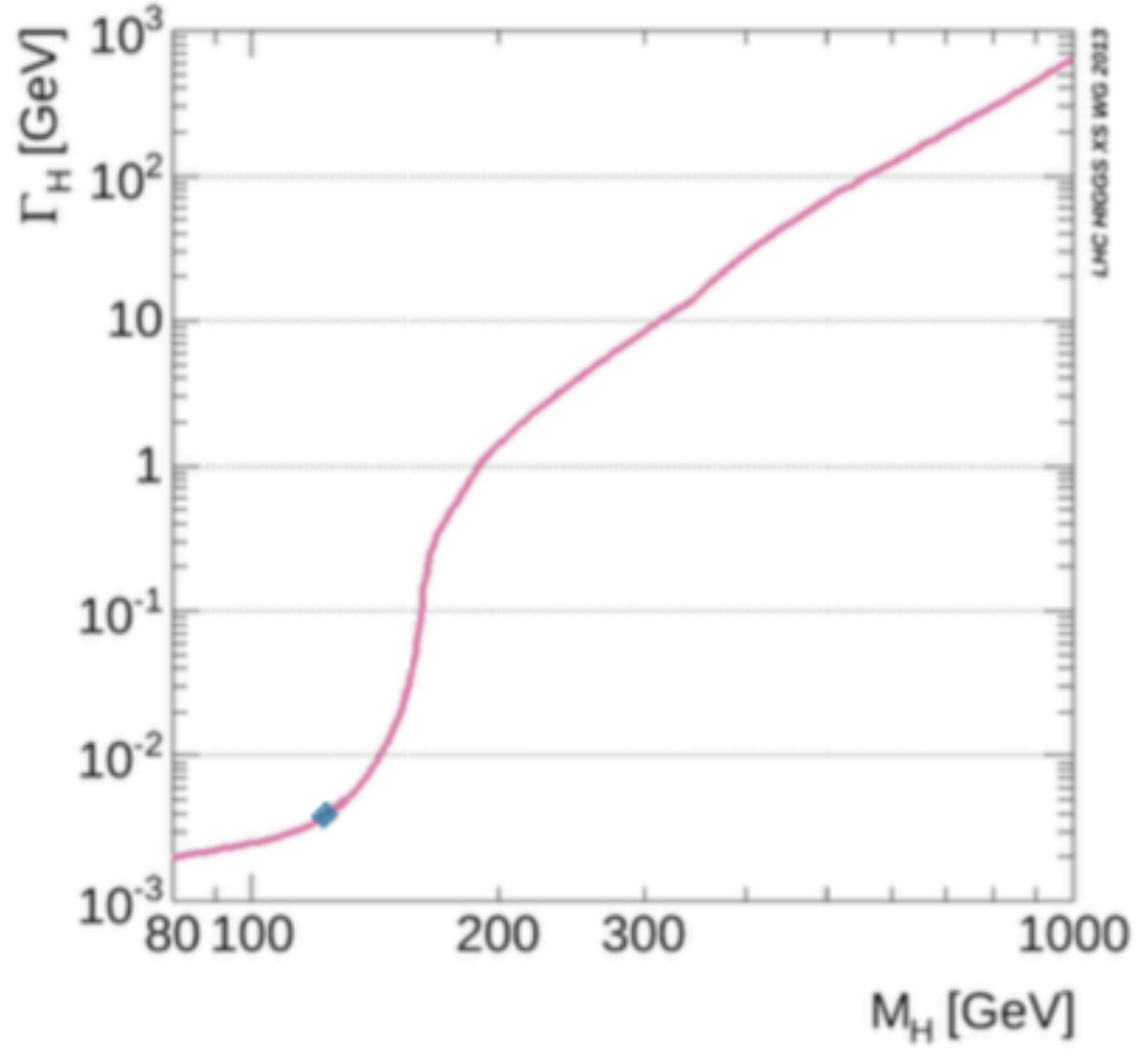
2. $M_H = 125$ GeV

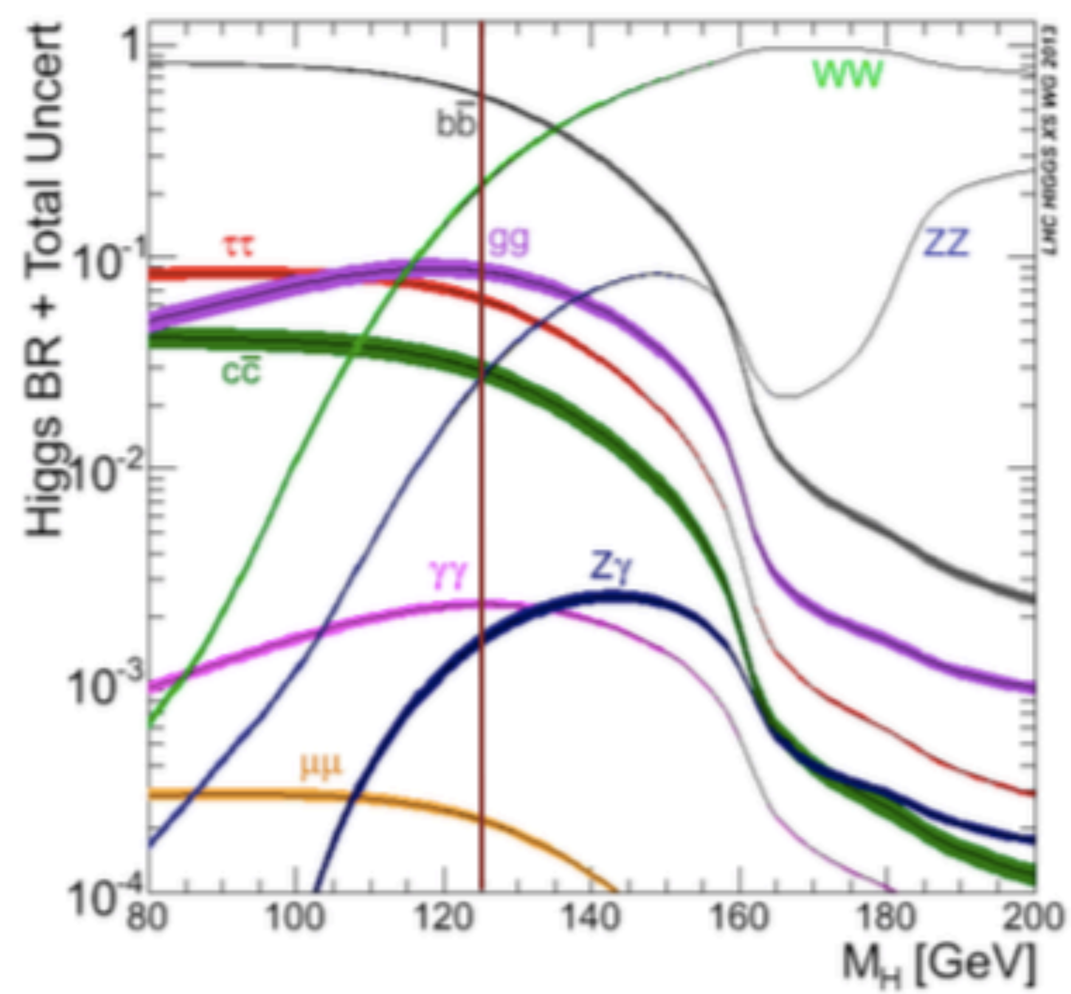
3. Width = 4 MeV

4. Weak couplings to SM particles “proportional" to the mass ⇒ it can radiated by heavy particles

5. QCD and electrically neutral ⇒ interactions with gluons and photons only through loops, it does not radiate.

Higgs decays





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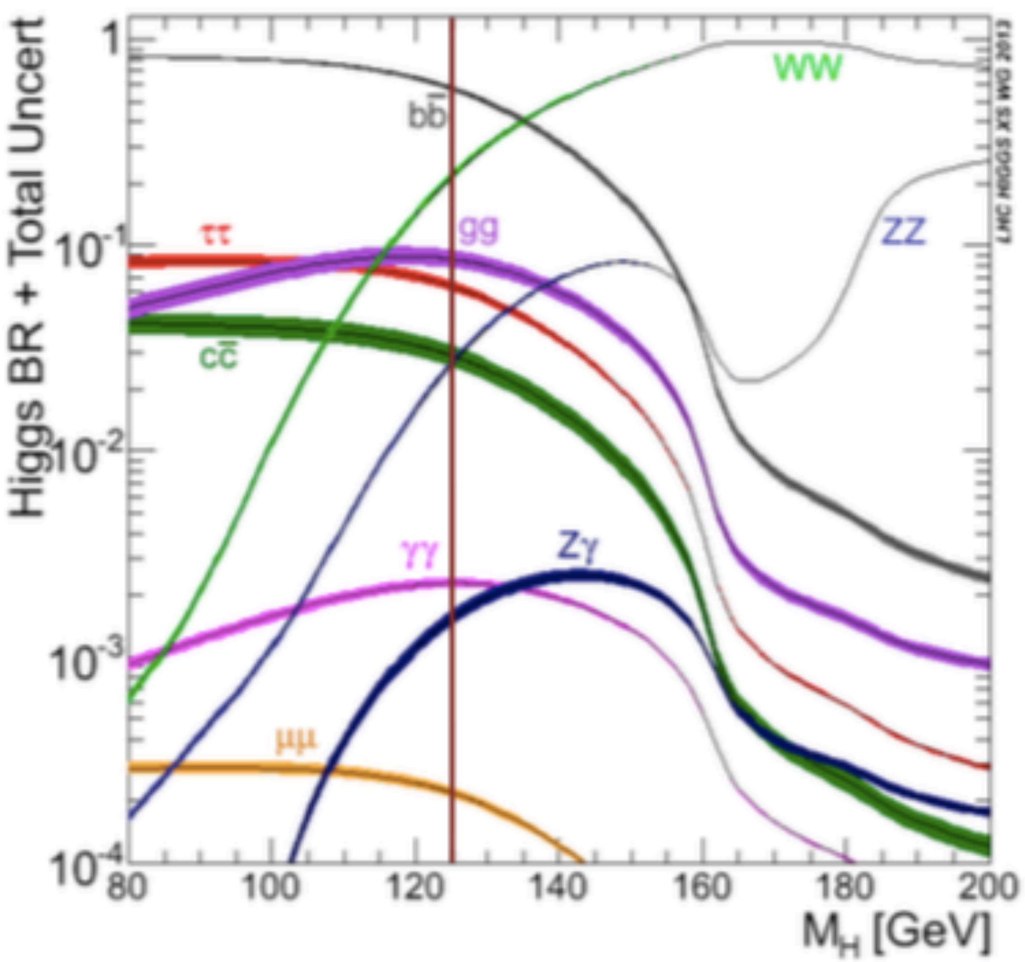


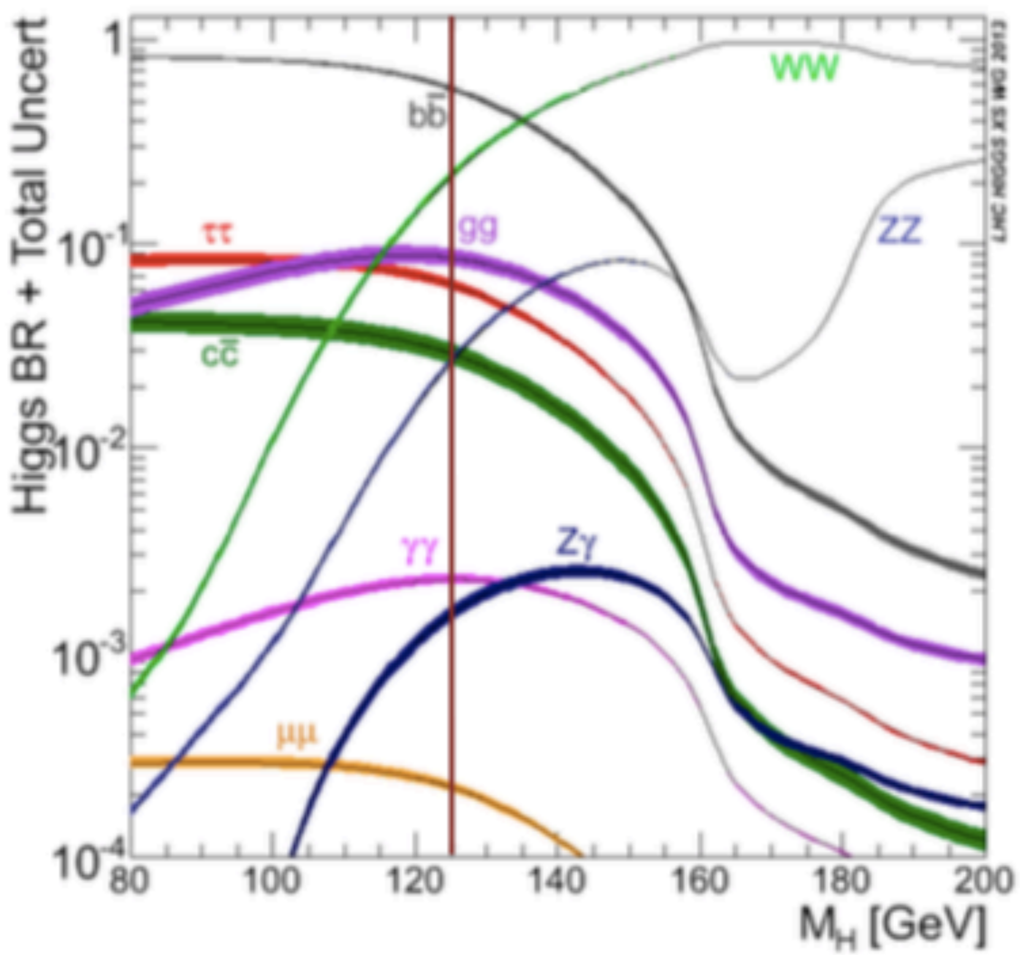
$$\Gamma(h \to f\bar{f}) = \frac{G_F m_f^2 N_{ci}}{4\sqrt{2}\pi} m_h \beta_F^3$$

$$\beta_F \equiv \sqrt{1 - 4m_f^2/m_h^2}$$

$$\Gamma(h \to q\bar{q}) = \frac{3G_F}{4\sqrt{2}\pi} m_q^2(m_h^2) m_h \beta_q^3 \left(1 + 5.67\frac{\alpha_s(m_h^2)}{\pi} + \cdots\right)$$

- H→bb dominating decay mode
- H→tau tau second most important one
- H→c c smaller because of the quark mass running!

Higgs decays



$$\Gamma(h \to WW^*) = \frac{3g^4 m_h}{512\pi^3} F\left(\frac{M_W}{m_h}\right)$$

$$\Gamma(h \to ZZ^*) = \frac{g^4 m_h}{2048\cos_W^4 \pi^3}\left(7 - \frac{40}{3}s_W^2 + \frac{160}{9}s_W^4\right) F\left(\frac{M_Z}{m_h}\right),$$

$$F(x) = -\,|\,1 - x^2\,|\left(\frac{47}{2}x^2 - \frac{13}{2} + \frac{1}{x^2}\right) + 3(1 - 6x^2 + 4x^4)\,|\ln x\,| + \frac{3(1 - 8x^2 + 20x^4)}{\sqrt{4x^2 - 1}}\cos^{-1}\left(\frac{3x^2 - 1}{2x^3}\right)$$

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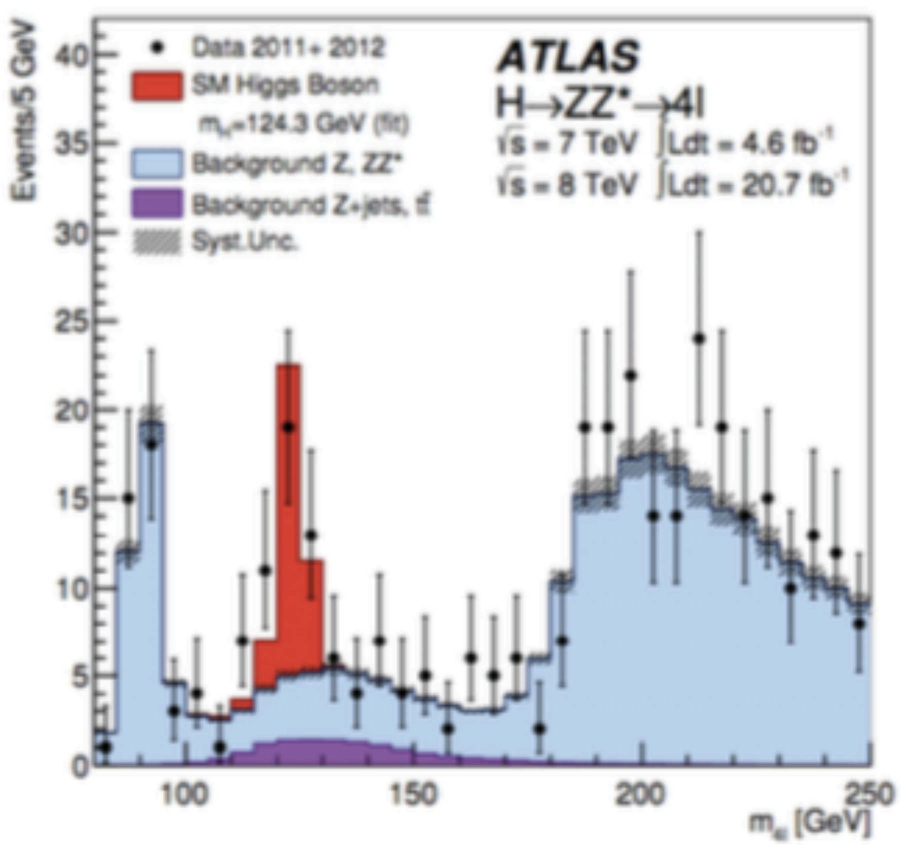

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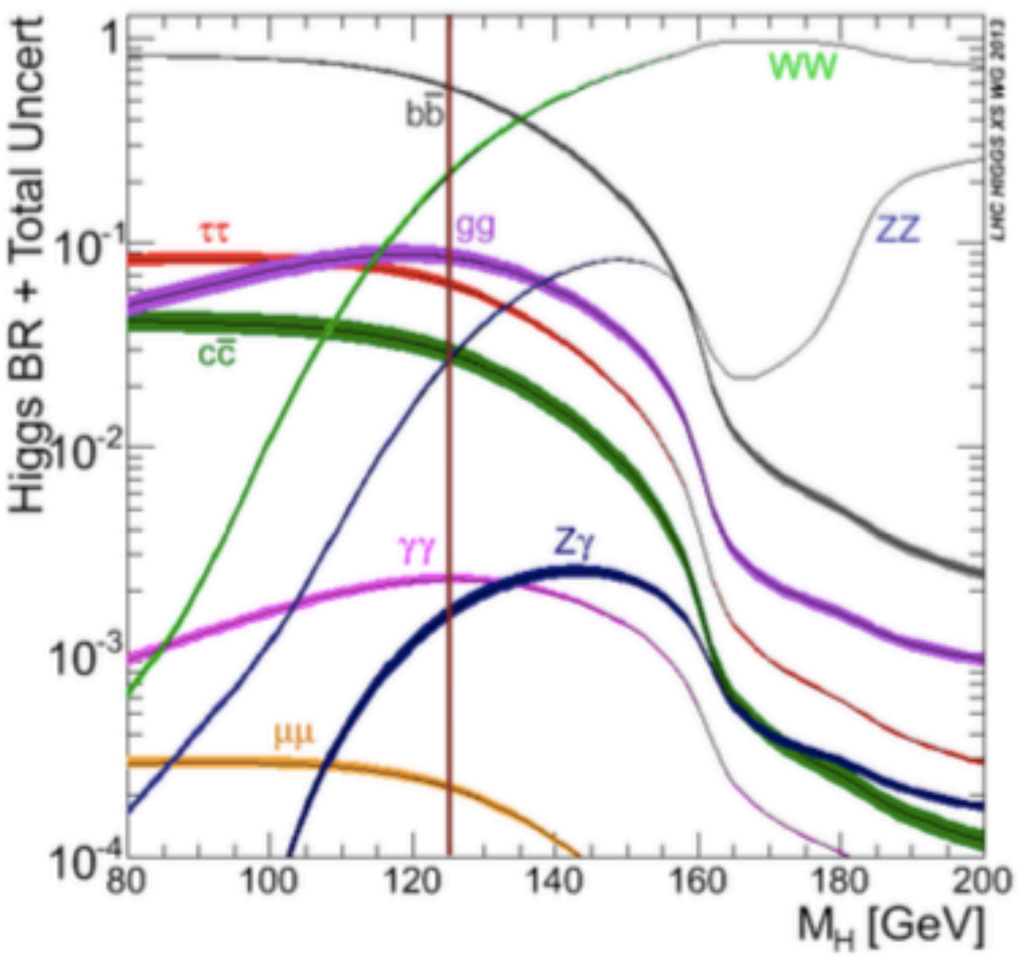
- 4l channel has been the discovery mode

Higgs decays



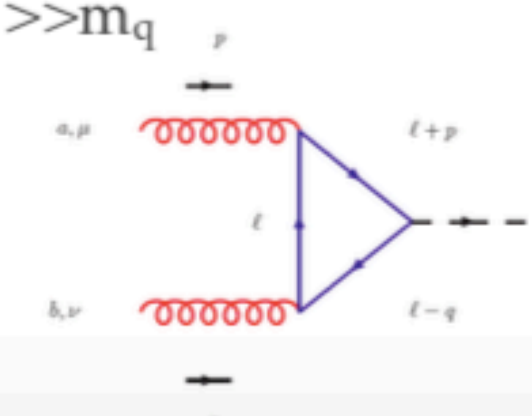
$$\Gamma(h \to gg) = \frac{G_F \alpha_s^2 m_h^3}{64\sqrt{2}\pi^3} \mid \sum_q F_{1/2}(\tau_q) \mid^2$$

where $\tau_q \equiv 4m_q^2/m_h^2$ and $F_{1/2}(\tau_q)$ is defined to be,

$$F_{1/2}(\tau_q) \equiv -2\tau_q \left[1 + (1-\tau_q) f(\tau_q)\right] .$$

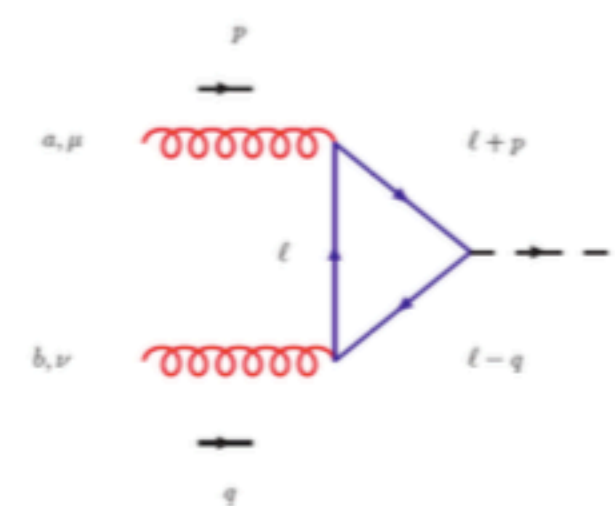
$$F_{1/2} \to \frac{2m_q^2}{m_h^2} \log^2\left(\frac{m_q}{m_h}\right) \quad \text{for } m_h >> m_q$$

$$F_{1/2} \to -\frac{4}{3}. \quad \text{for } m_q >> m_h$$



$H \to gg$ at one loop

In this case, this means that the loop calculation has to give a finite result!



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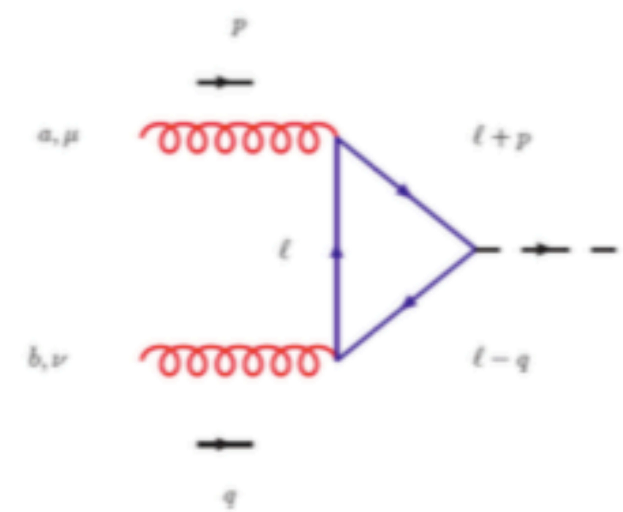

Let's do the calculation!

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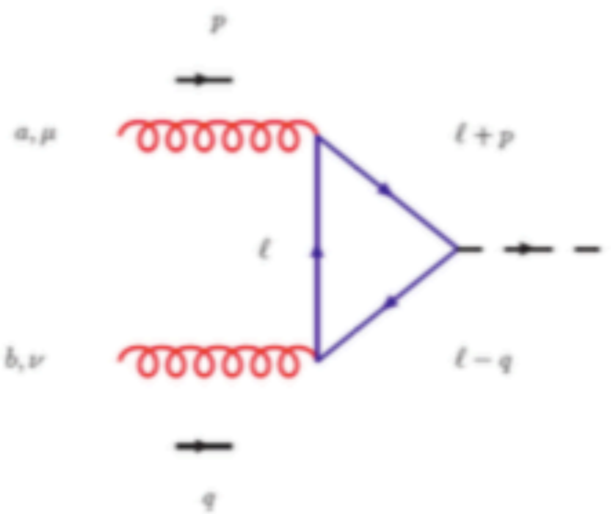

Let's do the calculation!

$$i\mathcal{A} = -(-ig_s)^2 \mathrm{Tr}(t^a t^b) \left(\frac{-im_t}{v} \right) \int \frac{d^d \ell}{(2\pi)^n} \frac{T^{\mu\nu}}{\mathrm{Den}} (i)^3 \epsilon_\mu(p) \epsilon_\nu(q)$$

where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell + p)^2 - m_t^2][(\ell - q)^2 - m_t^2]$$

H → gg at one loop

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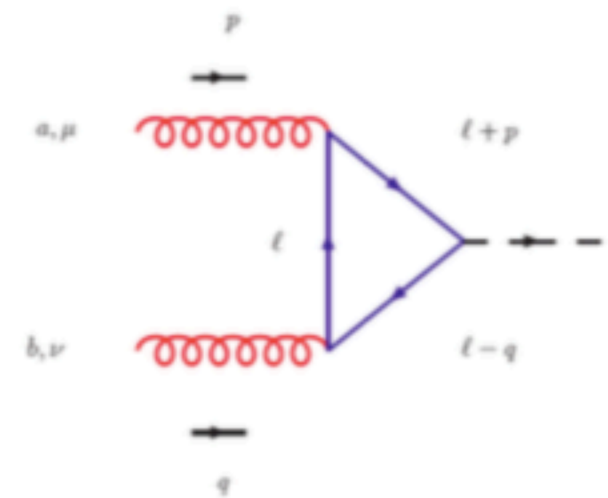

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where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

We combine the denominators into one by using $\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

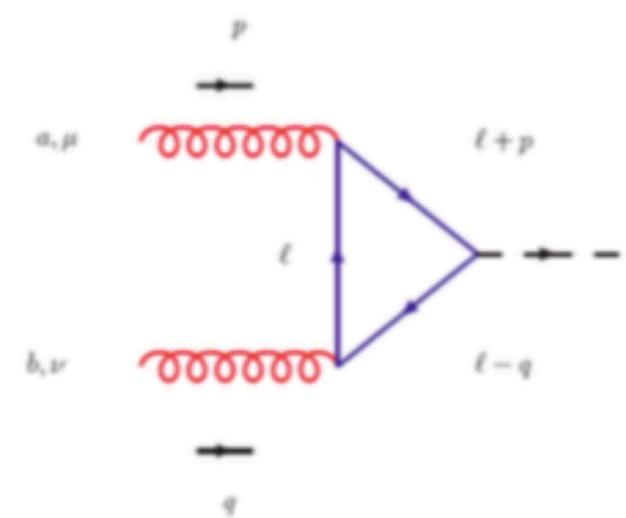
$$\frac{1}{\mathrm{Den}} = 2\int dx\, dy \frac{1}{[\ell^2 - m_t^2 + 2\ell\cdot(px - qy)]^3}.$$

$H \rightarrow gg$ at one loop

We shift the momentum:

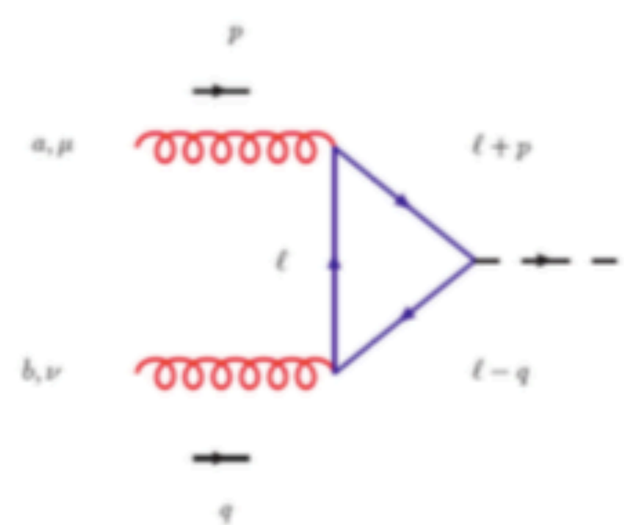
$$\ell' = \ell + px - qy$$

$$\frac{1}{\mathrm{Den}} \rightarrow 2\int dx\, dy \frac{1}{[\ell'^{\,2} - m_t^2 + M_H^2 xy]^3}.$$



H → gg at one loop

We shift the momentum:



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And now the tensor in the numerator:

$$T^{\mu\nu} = \text{Tr}\Big[(\ell + m_t)\gamma^\mu(\ell + p + m_t)(\ell - q + m_t)\gamma^\nu)\Big]$$

$$= 4m_t\Big[g^{\mu\nu}(m_t^2 - \ell^2 - \frac{M_H^2}{2}) + 4\ell^\mu\ell^\nu + p^\nu q^\mu\Big]$$

where I used the fact that the external gluons are on-shell. This trace is proportional to mt ! This is due to the spin flip caused by the scalar coupling.

$H \to gg$ at one loop

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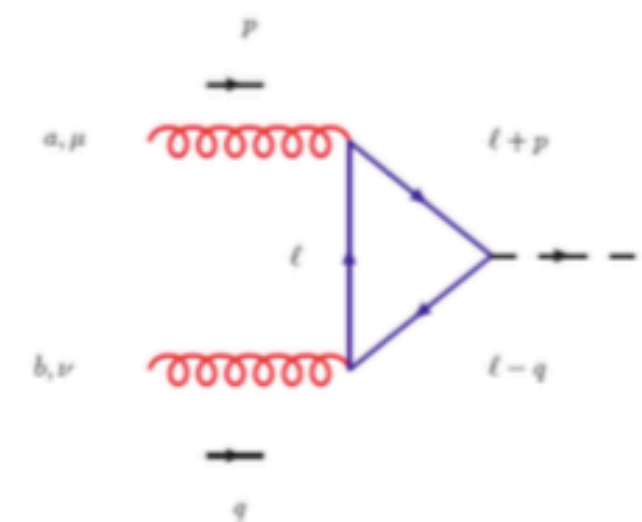

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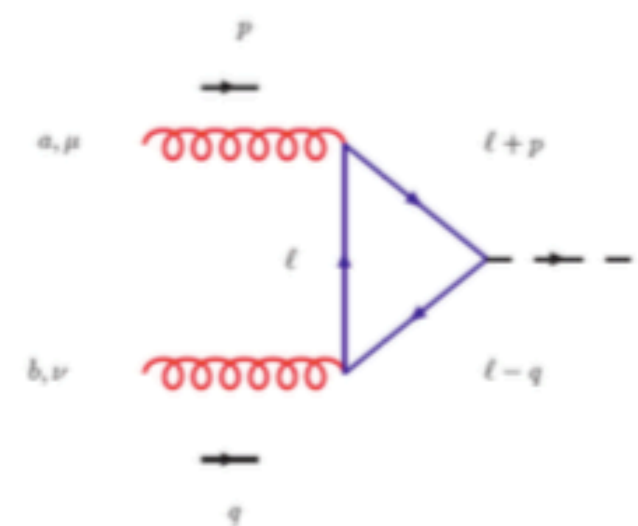
where I used the fact that the external gluons are on-shell. This trace is proportional to mt ! This is due to the spin flip caused by the scalar coupling.
Now we shift the loop momentum also here, we drop terms linear in the loop momentum (they are odd and vanish)

$H \rightarrow gg$ at one loop

We perform the tensor decomposition using:

$$\int d^d k \frac{k^\mu k^\nu}{(k^2 - C)^m} = \frac{1}{d} g^{\mu\nu} \int d^d k \frac{k^2}{(k^2 - C)^m}$$



So I can write an expression which depends only on scalar loop integrals:

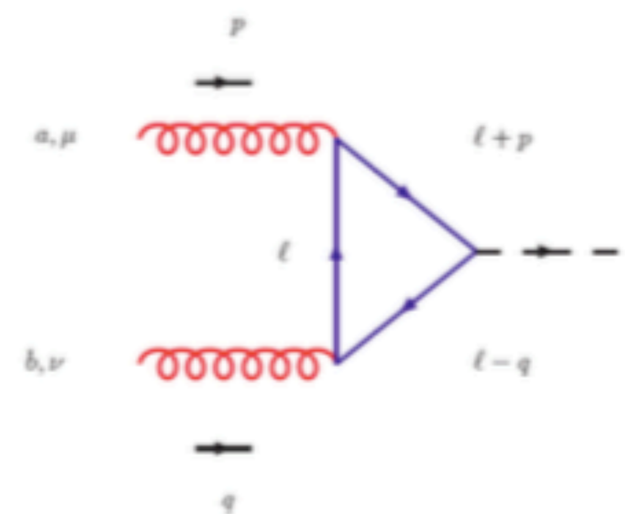
$$i\mathcal{A} = -\frac{2g_s^2 m_t^2}{v}\delta^{ab} \int \frac{d^d\ell'}{(2\pi)^d} \int dxdy \Big\{ g^{\mu\nu}\Big[m^2 + \ell'^2\Big(\frac{4-d}{d}\Big) + M_H^2(xy - \frac{1}{2})\Big]$$
$$+ p^\nu q^\mu (1 - 4xy)\Big\} \frac{2dxdy}{(\ell'^2 - m_t^2 + M_H^2 xy)^3} \epsilon_\mu(p)\epsilon_\nu(q).$$

There's a term which apparently diverges....??
Ok, Let's look the scalar integrals up in a table (or calculate them!)

$H \to gg$ at one loop

$$\int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-C)^3} = \frac{i}{32\pi^2}(4\pi)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon}(2-\epsilon)C^{-\epsilon}$$

$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-C)^3} = -\frac{i}{32\pi^2}(4\pi)^\epsilon \Gamma(1+\epsilon)C^{-1-\epsilon}.$$



where d=4-2eps. By substituting we arrive at a very simple final result!!

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dx dy \left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

Comments:

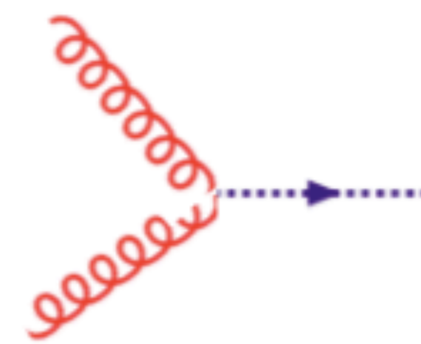
* The final dependence of the result is mt^2 : one from the Yukawa coupling, one from the spin flip.
* The tensor structure could have been guessed by gauge invariance.
* The integral depends on mt and mh.

Higgs effective coupling to gluons

Let's consider the case where the Higgs is light:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dx dy \left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



Higgs effective coupling to gluons

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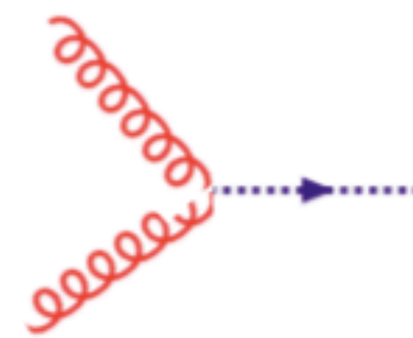


This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling). Any heavy quark coupled as in the SM to the Higgs boson gives the same contribution.

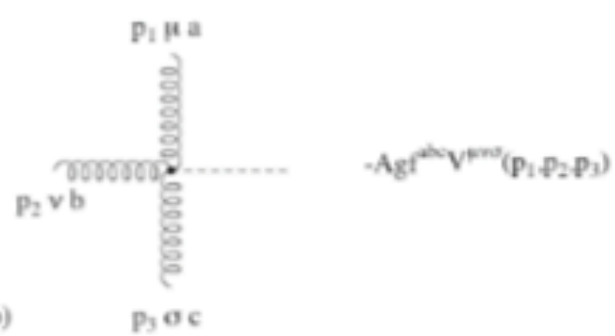
Higgs effective coupling to gluons

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(1 - \frac{\alpha_S}{3\pi}\frac{H}{v}\right) G^{\mu\nu} G_{\mu\nu}$$

This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.

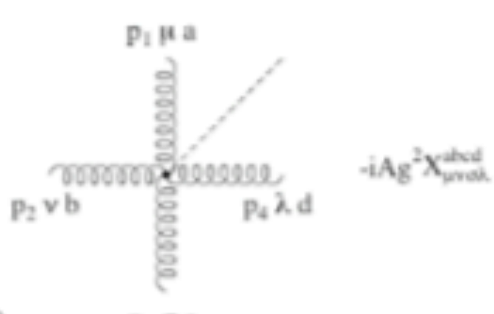
(a)

$$H^{\mu\nu}(p_1, p_2) = g^{\mu\nu} p_1 \cdot p_2 - p_1^\nu p_2^\mu .$$



(b)

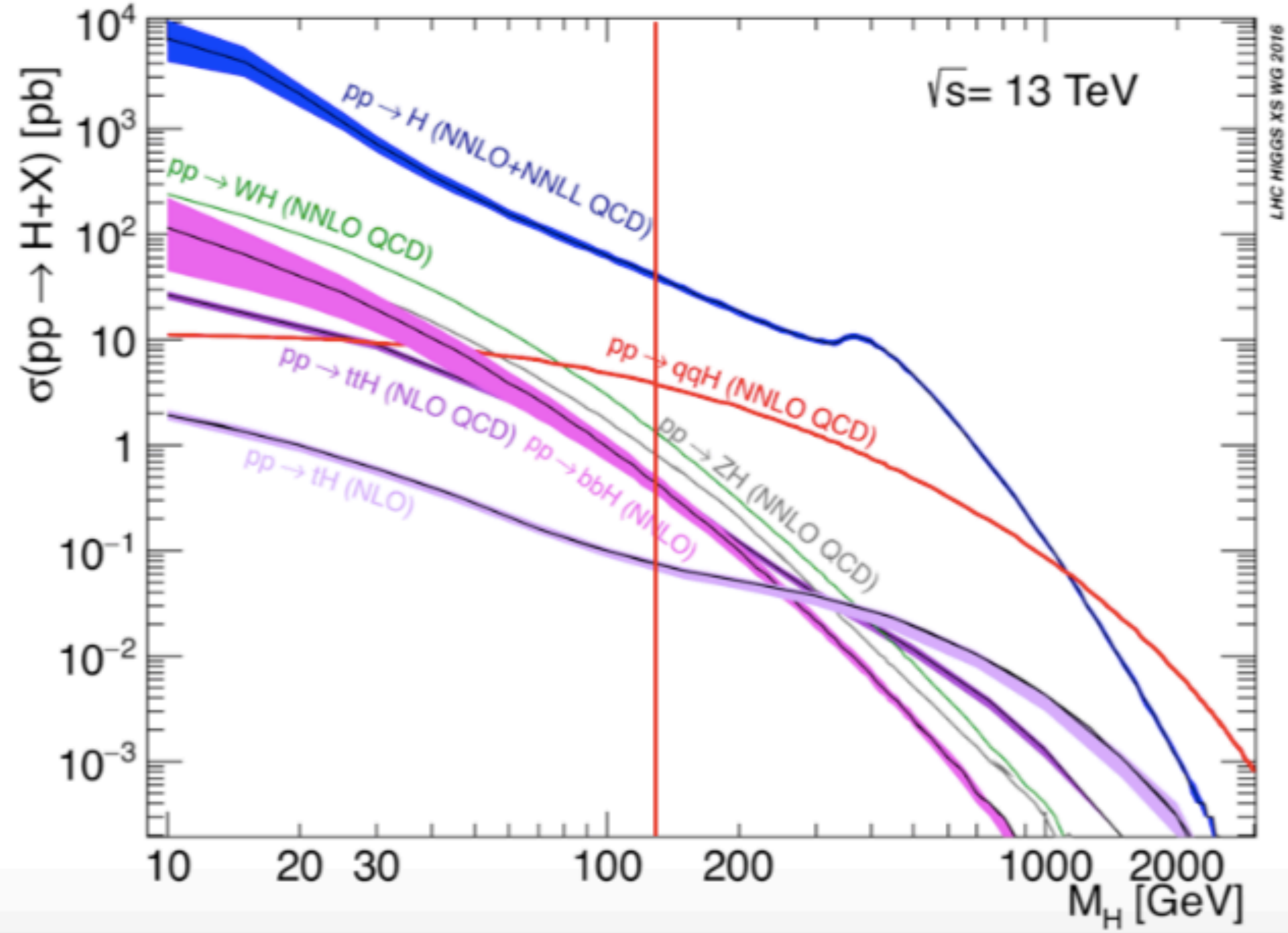
$$V^{\mu\nu\rho}(p_1, p_2, p_3) = (p_1 - p_2)^\rho g^{\mu\nu} + (p_2 - p_3)^\mu g^{\nu\rho} + (p_3 - p_1)^\nu g^{\rho\mu},$$



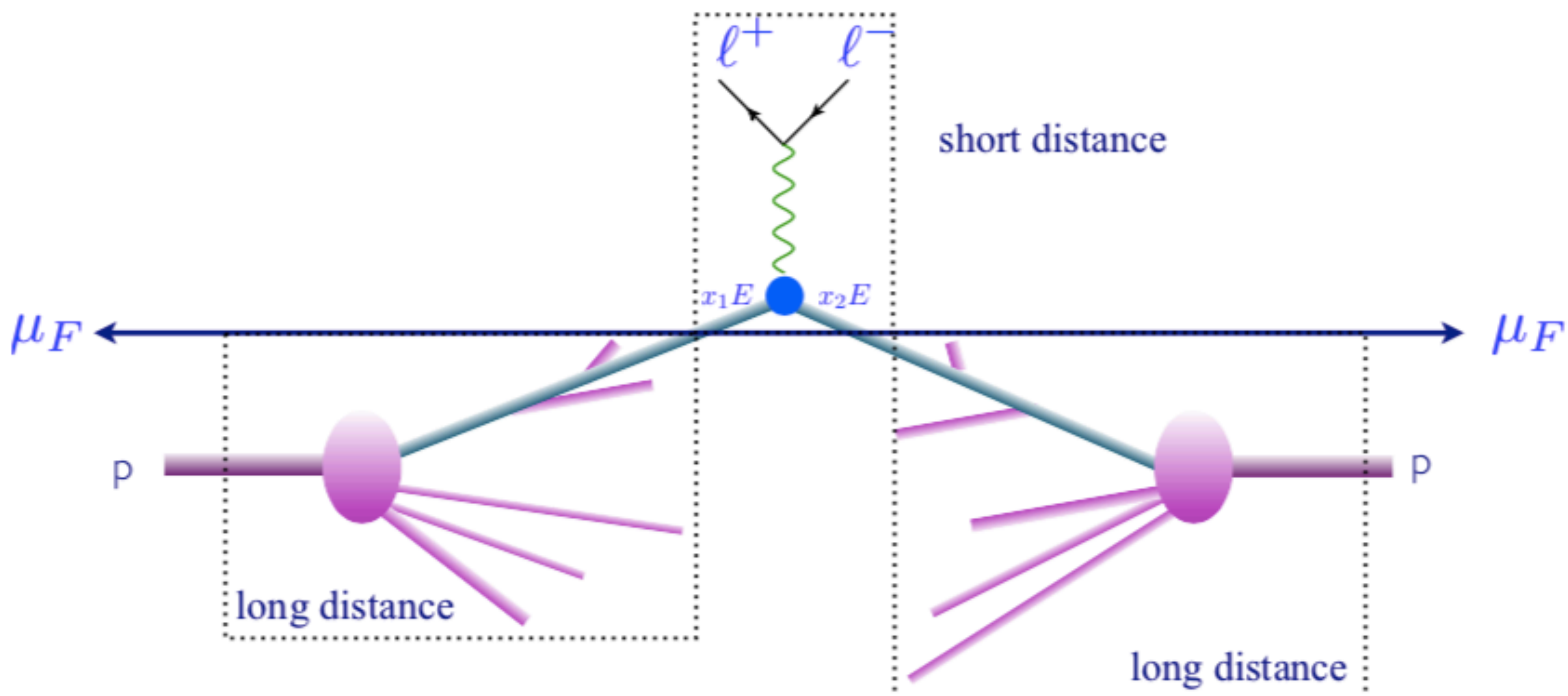
(c)

$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} &= f_{abe} f_{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &+ f_{ace} f_{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &+ f_{ade} f_{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) . \end{aligned}$$

Higgs production at the LHC



The LHC master formula



$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

The LHC master formula

$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

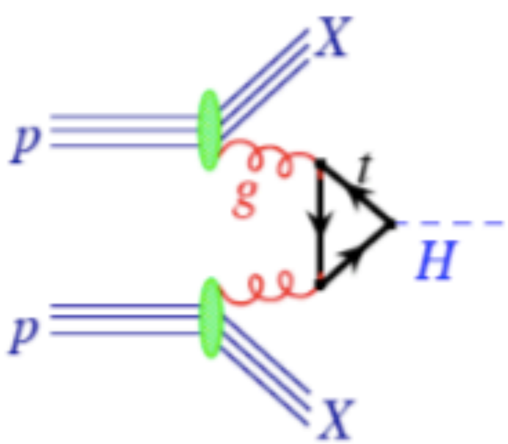
- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

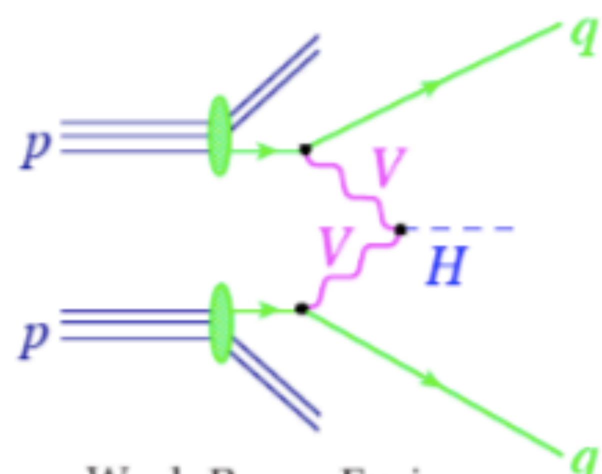
LO predictions | NLO corrections | NNLO corrections | NNNLO corrections

- Including higher corrections improves predictions and reduces theoretical uncertainties: improvement in accuracy and precision.

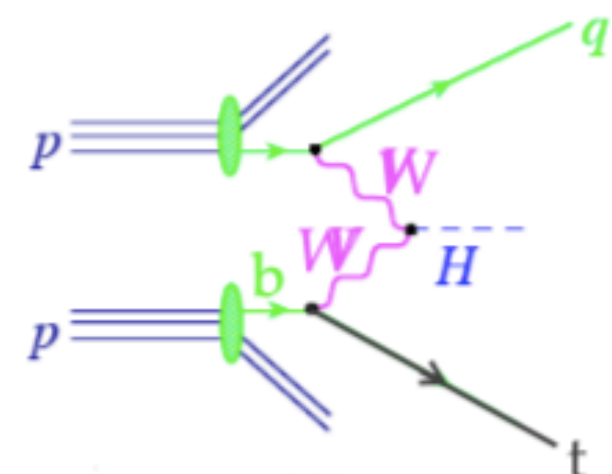
Higgs production channels



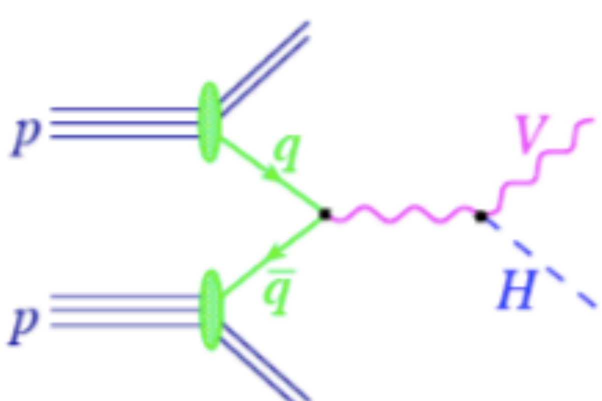
Gluon fusion



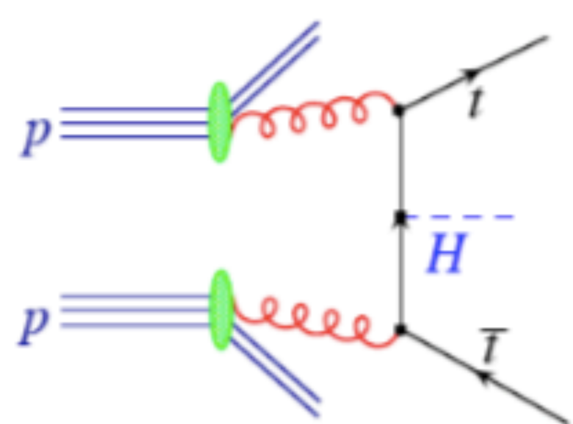
Weak-Boson Fusion



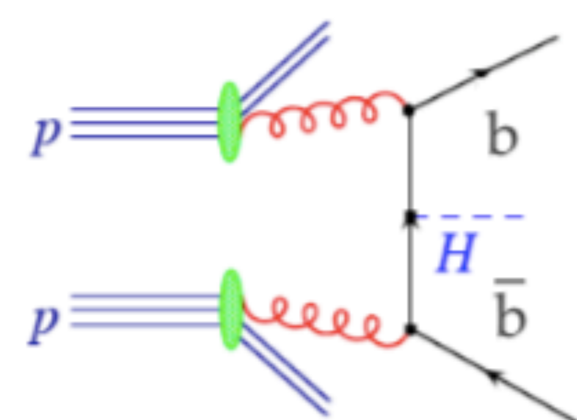
tHj



Higgs Strahlung

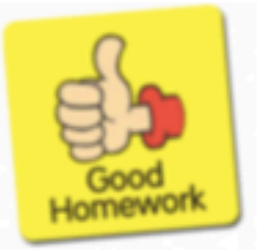


$t\bar{t}H$



bbH

pp→H at LO



$$\sigma^{\rm LO}(H+X) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 f_g(x_1,\mu_F) f_g(x_2,\mu_F) \times \hat{\sigma}^{(0)}(gg\to H)\,,$$

where $\tau_0 = m_H^2/S$ and $s = x_1 x_2 S$. $\hat{\sigma}$ for a $2\to 1$ process can be rewritten as

$$\begin{aligned}\hat{\sigma} &= \frac{1}{2s}\overline{|\mathcal{A}|^2}\frac{d^3P}{(2\pi)^3 2E_H}(2\pi)^4\delta^4(p+q-P_H)\\ &= \frac{1}{2s}\overline{|\mathcal{A}|^2}2\pi\delta(s-m_H^2)\,,\end{aligned}$$

where

$$\tau \equiv x_1 x_2 = \frac{S}{s}, \qquad \tau_0 = \frac{m_H^2}{S}\,.$$

Performing the change of variables $x_1, x_2 \to \tau, y$ with $x_1 \equiv \sqrt{\tau}e^{y}$, $x_2 \equiv \sqrt{\tau}e^{-y}$ (verify that the jacobian J is equal to 1) the change of the integration limits and the result becomes

$$\sigma^{\rm LO}(H+X) = \frac{\pi\overline{|\mathcal{A}|^2}}{m_H^2 S}\int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy\, xg(\sqrt{\tau_0}e^{y})g(\sqrt{\tau_0}e^{-y})\,.$$

pp→H at LO

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1,\mu_f) g(x_2,\mu_f)\, \hat{\sigma}(gg \to H)$$

$$= \frac{\alpha_S^2}{64\pi v^2} \mid I\left(\frac{M_H^2}{m^2}\right) \mid^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy g(\sqrt{\tau_0}e^{y}) g(\sqrt{\tau_0}e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

pp→H at LO

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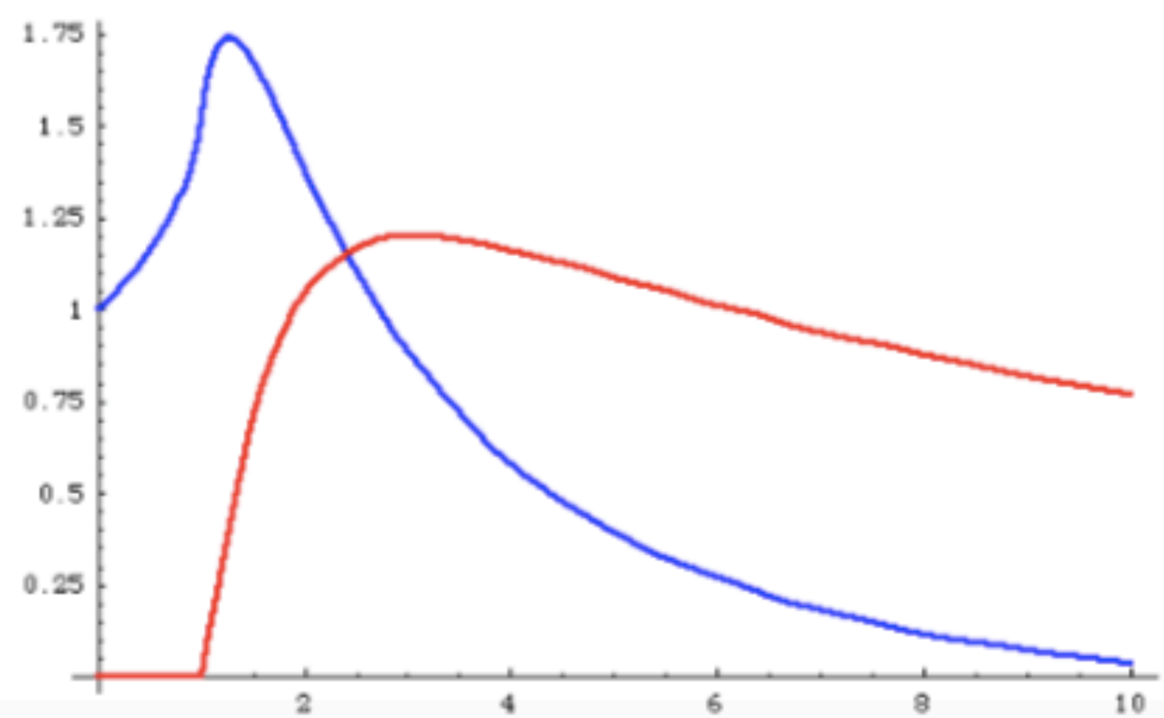
The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

I(x) has both a real and imaginary part, which develops at mh =2mt.



pp→H at LO

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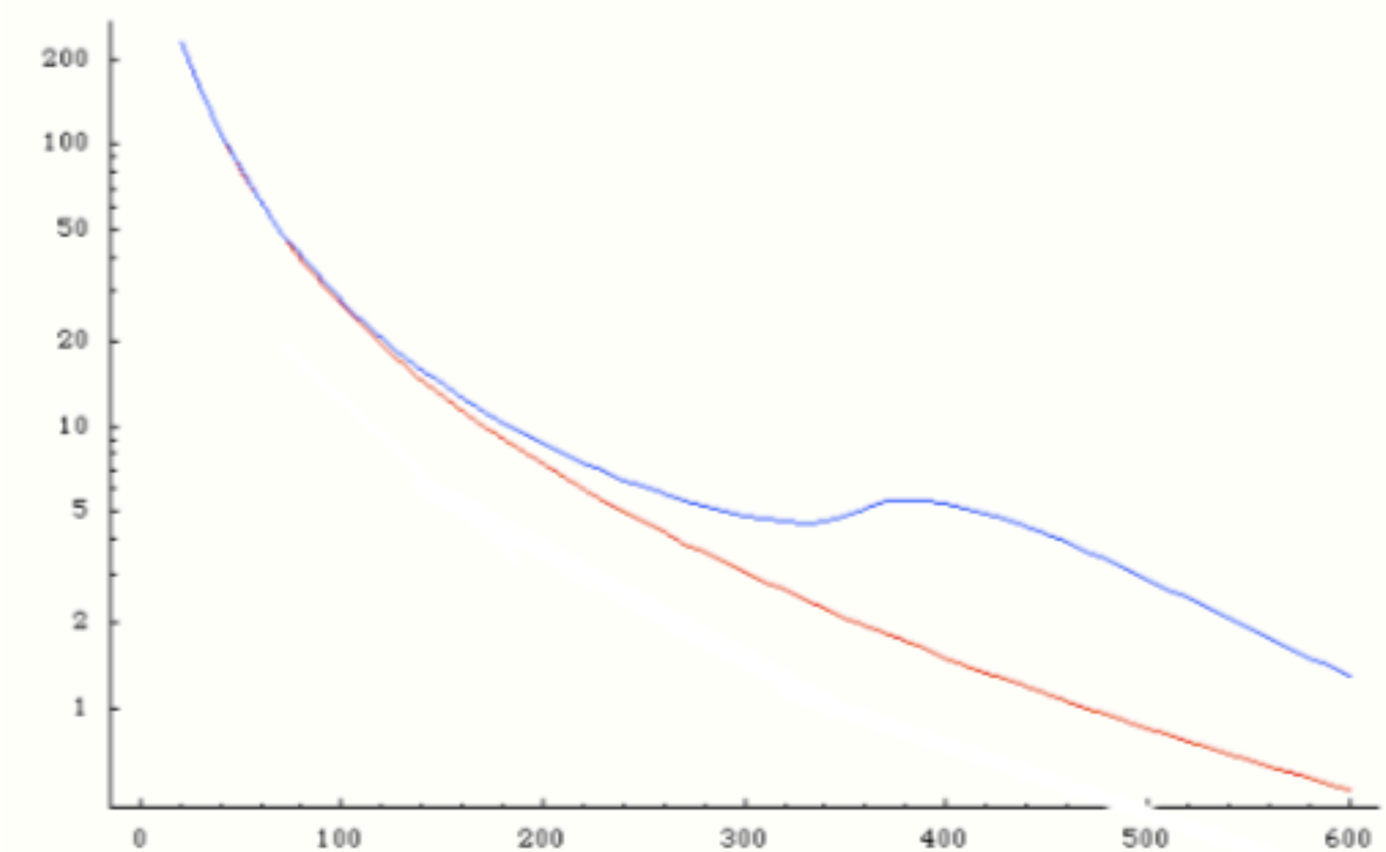
The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

I(x) has both a real and imaginary part, which develops at mh =2mt.

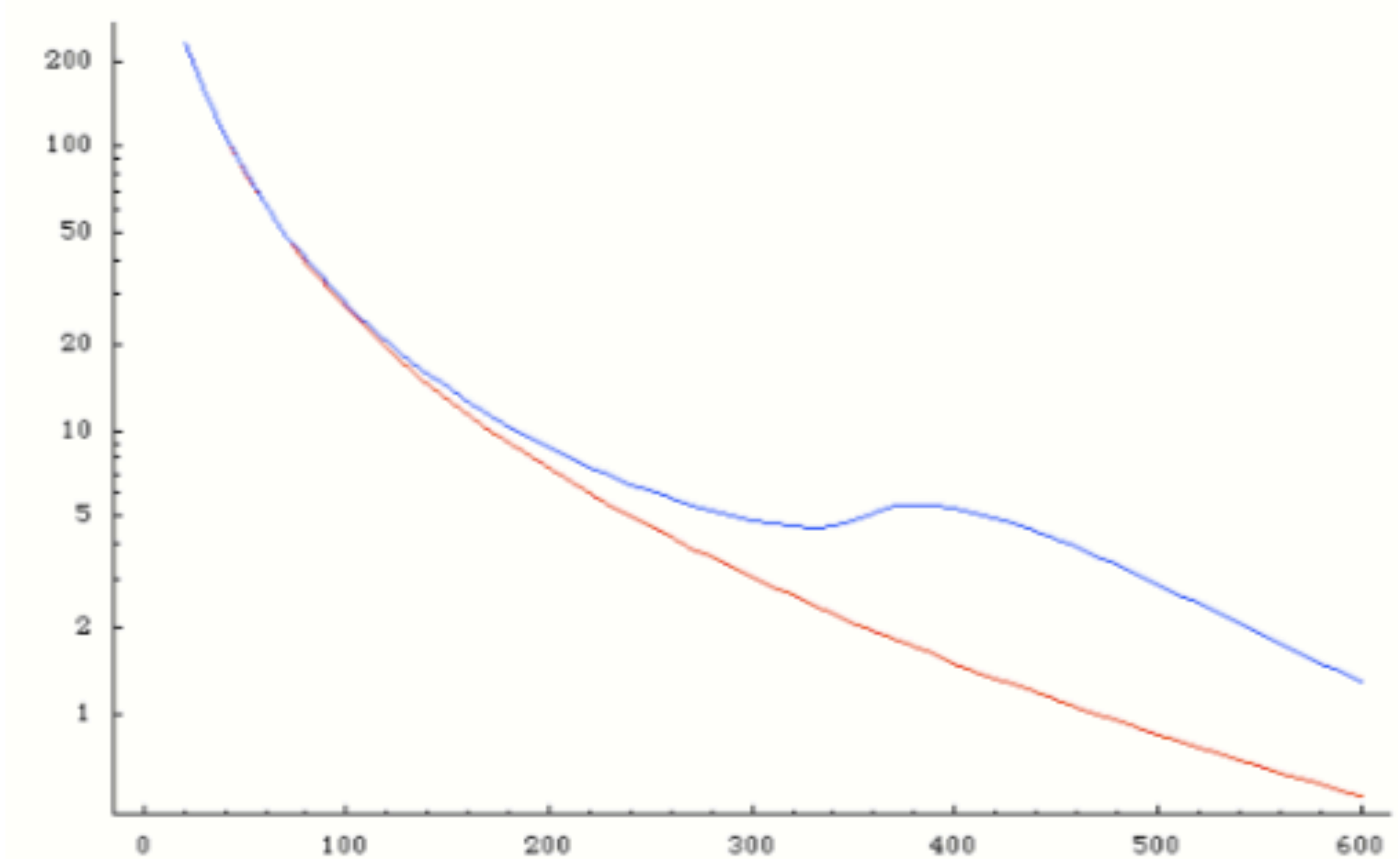
This causes a bump in the cross section.



pp→H at LO

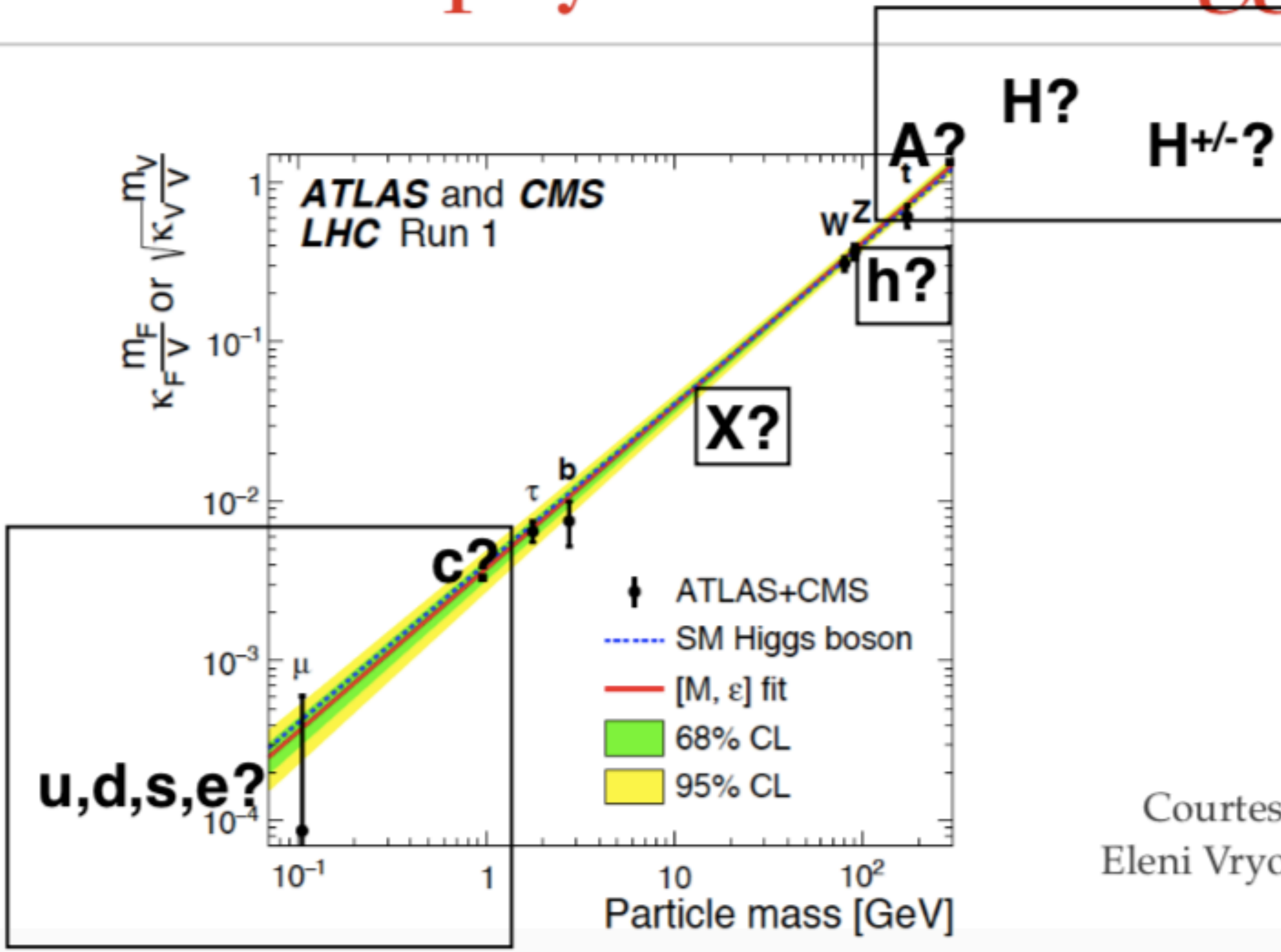
The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit m→∞.

For light Higgs is better than 10%.



So, if we are interested in a light Higgs we use the HEFT and simplify our life. If we do so, the NLO calculation becomes a standard 1-loop calculation, similar to Drell-Yan at NLO. We can (try to) do it!!

Search for new physics via the Higgs



Courtesy of
Eleni Vryonidou